[This question paper contains 2 printed pages.]

Your Roll No...............

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Sr. No. of Question Paper	**:**	**2675**
Unique Paper Code	:	2352571201
Name of the Paper	:	Elementary Linear Algebra
Name of the Course	:	**B.A. / B.Sc. (Programme) with Mathematics as Non-Major / Minor**
Semester	:	II – DSC

Duration : 3 Hours Maximum Marks : 90

<u>Instructions for Candidates</u>

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **any 5** parts from Question **1** and any **2** parts from Question **2-6**.

- Each Question carries **15** marks.
- Use of calculator is **NOT** allowed.

1. Attempt any 5 parts: : (**All parts carry 3 marks**)
   i) Are the vectors $\boldsymbol{x} = [\mathbf{-3, 1, 2}]$ and $\boldsymbol{y} = [\mathbf{6, -2, 4}]$ parallel? Justify.
   ii) Is $[5, -1, 2]$ a linear combination of $[-1, 0, 3]$ and $[2, -1, -1]$? Justify.
   iii) Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be vectors in $\mathbb{R}^n$, show that if $\|\boldsymbol{a}+\boldsymbol{b}\|^2 = \|\boldsymbol{a}\|^2 + \|\boldsymbol{b}\|^2$, then $\boldsymbol{a}.\boldsymbol{b} = \mathbf{0}$.
   iv) Let $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ be vectors in $\mathbb{R}^n$. Is it true that $\boldsymbol{x}.\boldsymbol{y} - \boldsymbol{x}.\boldsymbol{z} = \boldsymbol{x}.(\boldsymbol{y}-\boldsymbol{z})$? Justify.
   v) Find a unit vector in the direction of vector $[3, -5, 6]$.
   vi) $(7, -3, 6), (11, -5, 3), (10, -7, 8)$ are vertices of an isosceles triangle. True or False? Justify.
   vii) If two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ in $\mathbb{R}^n$ are parallel, then they have the same direction. True or False? Justify.

2. a. Find the quadratic equation $\mathrm{y} = \mathrm{ax}^2 + \mathrm{bx} + \mathrm{c}$ that passes through the points $(1, 0), (-1, 12)$ and $(2, 9)$.

   b. Use Gauss Jordan method to find complete solution set for the following homogeneous system of equations. Also give one particular solution.

$$\begin{aligned} 2x_1 - 6x_2 + 3x_3 - 21x_4 &= 0 \\ 4x_1 - 5x_2 + 2x_3 - 24x_4 &= 0 \\ -x_1 + 3x_2 - x_3 + 10x_4 &= 0 \\ -2x_1 + 3x_2 - x_3 + 13x_4 &= 0 \end{aligned}$$

   c. Find the rank of the matrix $B = \begin{bmatrix} -1 & 4 & 19 \\ 5 & 1 & -11 \\ 4 & -5 & -32 \\ 2 & 1 & -2 \\ 1 & -2 & -11 \end{bmatrix}$.

3. a. Find all eigen values of a matrix

$$\mathrm{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -3 \\ 0 & 0 & -5 \end{bmatrix}$$

   Also express each eigen space as a linear combination of fundamental eigen vectors.

b. Show that the set of vectors of the form [2a – 3b, a – 5c, a, 4c – b, c] in $\mathbb{R}^5$ forms a subspace of $\mathbb{R}^5$ under usual operations.

c. Prove or disprove :

i) The set of polynomials of degree 5 is a vector space under the operations of usual addition and scalar multiplication of polynomials.

ii) The set of singular 2*2 matrices is a vector space under the operations of usual addition and scalar multiplication of matrices.

4. a. Use simplified span method to find a simplified general form for all vectors in span(S), where
$$S = \left\{ \begin{bmatrix} -1 & 3 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} -2 & -5 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ -3 & 4 \end{bmatrix} \right\}$$
is a subset of M_{22}.

b. Use independence test method to determine whether the set
$$T = \{[2,-1,3],[4,-1,6],[-2,0,2]\}$$
is linearly independent or a linearly dependent subset of $\mathbb{R}^3$?

c. Determine whether the subset
$$S = \{ x^2 + x + 1, x^2 - 1, x^2 + 1\}$$
of P_2 forms a basis of P_2, the set of all polynomials of degree ≤ 2.

5. a. State True or False for the following statements. Give justification for your choice:

(i) A linear transformation T from vector space V to W, maps linearly independent subset S of V to a linearly independent subset of W. (3.5)

(ii) The function $L: \mathbb{R}^3 \to \mathbb{R}^3$ given by $L(x, y, z) = (x + 1, y - 2, z + 3)$ is a linear transformation. (4)

b. Prove that the inverse of a linear transformation L, (if it exists), is also a linear transformation.

c. Find the matrix for the linear transformation $T: P_3(\mathbb{R}) \to \mathbb{R}^3$ defined by
$$T(ax^3 + bx^2 + cx + d) = (d + c\,, 2b\,, a - d\,)$$
with respect to the standard ordered bases for vector spaces $P_3(\mathbb{R})$ and $\mathbb{R}^3(\mathbb{R})$.

6. a. Find the kernel and range of the transformation $T: P_3(\mathbb{R}) \to \mathbb{R}^3$ defined by
$$T(p(x)) = (p(-1), p(0), p(1))$$
Also, verify the Dimension Theorem for T.

b. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that performs a counterclockwise rotation through an angle of 30°. Find the matrix representation of T with respect to ordered basis $B = \{[1,1],[2,3]\}$ for $\mathbb{R}^2$.

c. Let $T: P_3(\mathbb{R}) \to P_2(\mathbb{R})$ be a linear transformation given by
$$T(p(x)) = p'(x)$$
Check whether T is an invertible linear transformation or not?