[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2689 **L**

Unique Paper Code : 2374571201

Name of the Paper : DSC: Statistical Methods

Name of the Course : **B.A. / B.Sc. (P)**

Semester : II (NEP)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **Six** questions. **All** questions carry equal marks.
3. Use of a non-programmable scientific calculator is allowed.

1. (a) Define a random variable. Define a cumulative distribution function. For a discrete random variable, show that:

   i) $P(a < X \le b) = F(b) - F(a)$ and

   ii) $P(a \le X < b) = F(b) - F(a) - P(X = b) + P(X = a)$ (8)

   (b) A continuous random variable has the following probability density function:

   $$f(x) = A\,x^2, \quad 0 \le x \le 1.$$

   Determine: i) The value of A,

   ii) $P(0.2 \le X \le 0.5)$, and

   iii) $P\left(X > \frac{3}{4} \mid X > \frac{1}{2}\right)$. (7)

2. (a) Define the mathematical expectation of a random variable. Let X be a random variable with the following probability distribution:

X	-3	6	9
P(X=x)	$1/6$	$1/2$	$1/3$

   Find $E(X)$, $E(X^2)$, variance and hence evaluate $E[(2X+1)^2]$. (8)

   (b) Define the moment generating function (MGF), characteristic function, and cumulant generating function. Show that: $\frac{d}{dt} M_X(t)\Big|_{t=0} = E(X^r)$. (7)

3. (a) The joint probability distribution of two random variables X and Y is given below:

X\Y	1	2	3
1	0.15	0.20	0.15
2	0.15	0.20	0.15

   Find: i. E(X) and E(Y)

   ii. Var(X) and Var(Y)

   iii. P(X|Y=2) and

   iv. Check whether X and Y are independent (8)

   (b) The joint PDF of a two-dimensional random variable (X, Y) is given by

   $$f(x,y) = \begin{cases} \frac{x^3y^3}{16}, & 0 \le x \le 2,\ 0 \le y \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

P.T.O.

Find the marginal PDF of X and Y. Check whether the two random variables are independent. (7)

4. (a) A fair coin is tossed 10 times. Find:
    i. Probability of exactly 2 heads
    ii. Mean and variance
    iii. The mode(s) of the distribution and
    iv. Comment on the skewness of the distribution. (8)

   (b) Derive the recurrence relation for the probability mass function (PMF) of a binomial distribution. (4)

   (c) If X~Binomial (4,0.3), and Y~Binomial (6,0.3), find the $P(X + Y = 3)$. (3)

5. (a) Derive the cumulant generating function of a Poisson distribution. Using it, obtain β_1 and β_2. Hence, comment on the distribution's skewness. (8)

   (b) Show that the sum of independent Poisson random variables is Poisson. Explain why their difference does not follow a Poisson distribution. (7)

6. (a) Derive the moment generating function of a normal distribution. Using it, show that the odd-order standard moments are zero. (8)

   (b) If X is a normal variate with mean 10 and S.D. 3, find the probability that
    i. $7 \leq X \leq 13$
    ii. $|X - 15| > 4$

   Also, find the value of 'a' such that $P(X < a) = 0.25$. (7)

7. (a) Define exponential distribution. Obtain the mean and variance of the distribution. If X has an exponential distribution, then for every constant $a \geq 0$, show that $P(Y \leq x|X \geq a) = P(X \leq x)$, for all x,
   where $Y = X - a$. (8)

   (b) Define the Gamma distribution and derive its cumulant generating function. Also, explain its relationship with the exponential distribution. (7)

8. (a) "The mean of the binomial random variable is less than its variance", comment. (5)

   (b) If X and Y are two independent normal random variables with parameters $N(6, 4)$ and $N(5,9)$, respectively. Find the distribution of $5X - 2Y$. (5)

   (c) If X is Poisson (λ) variate such that

$$P(X = 2) = 9\ P(X = 4) + 90\ P(X = 6),$$

   Find:
   (i) λ.
   (ii) mean of X.
   (iii) coefficient of skewness. (5)