[This question paper contains 2 printed pages.]

Your Roll No................

Sr. No. of Question Paper	:	**3079** **L**
Unique Paper Code	:	2352571201
Name of the Paper	:	Elementary Linear Algebra
Name of the Course	:	**B.A. / B.Sc. (Programme) with Mathematics as Non-Major / Minor**
Semester	:	II – DSC

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **any 5** parts from Question **1** and any **2** parts from Question **2-6**.

- Each Question carries **15** marks.
- Use of calculator is **NOT** allowed.

1. Attempt any 5 parts: : (**All parts carry 3 marks**)
   i). Are the vectors $\boldsymbol{x} = [\mathbf{3}, -\mathbf{5}, \mathbf{2}]$ and $\boldsymbol{y} = [\mathbf{6}, -\mathbf{10}, \mathbf{5}]$ parallel? Justify.
   ii). Is $[-1, 3, 2]$ a linear combination of $[1, -2, 5]$ and $[0, 3, 1]$? Justify.
   iii). Prove that if $(\boldsymbol{a} + \boldsymbol{b}).(\boldsymbol{a} - \boldsymbol{b}) = 0$, then $\|\boldsymbol{a}\| = \|\boldsymbol{b}\|$.
   iv). Let $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ be vectors in $\mathbb{R}^n$. Is it true that $\boldsymbol{x}.\boldsymbol{y} + \boldsymbol{x}.\boldsymbol{z} = \boldsymbol{x}.(\boldsymbol{y} + \boldsymbol{z})$**?** Justify.
   v). Find a unit vector in the direction of vector $[6, 2, 0, -3]$.
   vi). $(2, 5, -3), (-1, 3, -2), (-3, -2, -1)$ are vertices of an isosceles triangle. True or False? Justify.
   vii). Let $\boldsymbol{x}, \boldsymbol{y}$ be vectors in $\mathbb{R}^n$. Is it true that $\boldsymbol{x}.\boldsymbol{y} = \mathbf{0}$, but neither $\boldsymbol{x} = \mathbf{0}$ nor $\boldsymbol{y} = \mathbf{0}$? Justify.

2. a. Find the quadratic equation $y = ax^2 + bx + c$ that passes through the points $(3, 25), (-1, 1)$ and $(-2, 0)$.

   b. Use Gauss Jordan method to find complete solution set for the following homogeneous system of equations. Also give one particular solution.

$$2x_1 + 6x_2 + 13x_3 + x_4 = 0$$
$$x_1 + 4x_2 + 10x_3 + x_4 = 0$$
$$2x_1 + 8x_2 + 20x_3 + x_4 = 0$$
$$3x_1 + 10x_2 + 21x_3 + 2x_4 = 0$$

   c. Find the rank of the matrix $Y = \begin{bmatrix} 3 & 5 & 2 \\ 4 & 2 & 3 \\ -1 & 2 & 4 \end{bmatrix}$.

3. a. Find the characteristic polynomial and eigen values of the matrix

$$A = \begin{bmatrix} 4 & 0 & -2 \\ 6 & 2 & -6 \\ 2 & 0 & 1 \end{bmatrix}$$

   Also find the eigen spaces corresponding to each eigen value of A.

   b. Check whether $\mathbb{R}$, the set of real numbers is a

   i) vector space with ordinary addition but with scalar multiplication replaced by

$$a \odot x = 0 \quad \text{for every real number } a.$$

ii) vector space with usual scalar multiplication but with vector addition replaced by $$x \oplus y = 2(x+y) \quad \text{for all } x, y \in \mathbb{R}.$$

c. Prove or disprove that the subset $S = \{p(x) \in P_5 \mid p'(4) = 0,$ where $p'(x)$ is the derivative of $p(x)\}$ of P_5 is a subspace of P_5 under usual operations, where P_5 is a set of all polynomials of degree ≤ 5.

4. a. Let S be a non empty subset of a vector space V. Then show that
   i) $Span(S)$ is a subspace of V.
   ii) If W is a subspace of V with $S \subseteq W$, then $Span(S) \subseteq W$.

   b. Show that $S = \{[2,-1,0,5],[1,-1,2,0],[-1,0,1,1]\}$ is a linearly independent subset of $\mathbb{R}^4$.

   c. Let W be the solution set to the matrix equation $AX = 0$, where
   $$A = \begin{bmatrix} -1 & -2 & -4 & 8 \\ 3 & 4 & 6 & -14 \\ -2 & -1 & 1 & 7 \\ 1 & 0 & -2 & 0 \\ 2 & 3 & 5 & -12 \end{bmatrix}$$
   Show that $dim\, W + rank\,(A) = 4$.

5. a. Check if the following mappings are linear transformation or not. Prove or disprove it with a suitable justification:
   (i) $T_1: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T_1(x,y,z) = (x+y, y-z, 1+z)$
   (ii) $T_2: P_3(\mathbb{R}) \to \mathbb{R}$ defined by $T_2(ax^3+bx^2+cx+d) = a+b+c+d$

   b. Find the matrix of the linear transformation : $P_3(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ defined by
   $$L(ax^3+bx^2+cx+d) = \begin{bmatrix} 3a-2c & -b+4d \\ 4b-c+3d & -6a-b+2d \end{bmatrix}$$
   with respect to standard ordered bases of $P_3(\mathbb{R})$ and $M_{2\times 2}(\mathbb{R})$, respectively.

   c. Does there exist a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1,0,3) = (1,1)$ and $T(-2,0,-6) = (2,1)$. Justify.

6. a. Let $T: P_2(\mathbb{R}) \to P_3(\mathbb{R})$ be the linear transformation defined by
   $$T(f(x)) = 2f'(x) + \int_0^x 3f(t)dt$$
   Find dimension of range space of T and kernel of T.

   b. Consider the linear operator $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by
   $$T(x,y) = (x\cos\alpha - y\sin\alpha, x\sin\alpha + y\cos\alpha)$$
   where α is a fixed angle. Then check that T is an isomorphism or not.

   c. Let $T: V \to W$ be a linear transformation, where V and W are finite dimensional vector spaces. Prove that T is onto if and only if $dim(Range(T)) = dim(W)$.