$$L\left(ax^3+bx^2+cx+d\right)=\begin{bmatrix} -3a-2c & -b+4d \\ 4b-c+3d & -6a-b+2d \end{bmatrix}$$

with respect to standard ordered bases of P_3 and $M_{2\times 2}(\mathbb{R})$.

Also compute $L\left(5x^3-x^2+3x+2\right)$ using this matrix.

6. (a) Let $T:\mathbb{R}^3\to\mathbb{R}^2$ be the liner transformation given by

$$T([x_1,x_2,x_3])=[-2x_1+3x_3, x_1+2x_2-x_3]$$

Find the matrix of T with respect to the ordered bases B = {[1,–3,2], [–4,13,–3], [2,–3,20]} for $\mathbb{R}^3$ and $C=\{[-2,-1],[5,3]\}$ for $\mathbb{R}^2$.

(b) Consider the linear transformation $T:P_2\to P_3$ given by $T\left(ax^2+bx+c\right)=cx^3+bx^2+ax$

Find a basis for ker(T) and range(T). Also, verify the Dimension theorem.

(c) Consider the linear transformation $T:\mathbb{R}^2\to\mathbb{R}^3$ given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)=\begin{bmatrix} -3 & 4 \\ -6 & 9 \\ 7 & -8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Show that L is one-to-one but not onto.

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This question paper contains 4 printed pages]

Your Roll No.	:	....
Sl. No. of Q. Paper	:	**6549** I
Unique Paper Code	:	2354001202
Name of the Paper	:	Introduction to linear Algebra
Name of the Course	:	**Common Prog Group**
Semester	:	II

Time : 3 Hours ***Maximum Marks : 90***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **all** question by selecting **two** parts from each question.

(c) **All** questions carry equal marks.

1. (a) If a and b are unit vectors in $\mathbb{R}^n$.Then prove that $-1\le a.b\le 1$.

(b) Solve the following system of linear equations using Gaussian-Elimination method. Examine the consistency of the system and if consistent, find the complete solution set.

2x–y–z=2

x+2y+z = 2

4x–7y–5z = 2

(c) Solve the following system of linear equations using Gauss-Jordan method:

x–y–2z = 0

3x+y+3z=0

2x+3y–z = 0

2. (a) Define rank of a matrix. Find the rank of following matrix by reducing into its reduced row echelon form: $A=\begin{bmatrix}1 & 3 & -1 & 4\\ 2 & 4 & 3 & 5\\ -1 & -2 & 6 & -7\end{bmatrix}$.

(b) Find the characteristic polynomial of the matrix: $A=\begin{bmatrix}5 & 4\\ 1 & 2\end{bmatrix}$. Also, find all the eigenvalues and eigenspaces for A.

(c) Let $A=\begin{bmatrix}2 & 1 & 0\\ 0 & 2 & 1\\ 0 & 0 & 2\end{bmatrix}$, Show that matrix A is not diagonalizable.

3. (a) Let V be a vector space over $\mathbb{R}$. Show that:

(i) $a0_V = 0_V$

(ii) If $av = 0_V$ then either a = 0 or $v = 0_V$

where $a \in \mathbb{R}, v \in V$ and 0_V denotes the zero vector of V.

(b) Define a subspace. Show that the set $S=\{p \in p_5 \mid p(3)=0\}$ is a subspace of p_5 under the usual operations.

(c) Examine whether the set S= {[1,2,1] , [2,3,1], [-1,2,-3]} forms a basis for $\mathbb{R}^3$ or not.

4. (a) Define linear independence and linear dependence for a finite set of vectors. Determine whether the set S = {[1,9,–2], [3,4,5], [–2,5,–7]} is linearly independent or not.

(b) Find a basis and dimension of the subspace W of $\mathbb{R}^3$, where $W=\{X \in \mathbb{R}^3 \mid AX=0\}$ and $A=\begin{bmatrix}2 & 1 & 4\\ 0 & 1 & -2\\ 0 & -2 & 1\end{bmatrix}$.

(c) Use the simplified span method to check whether the set {[1,3,–1], [2,7,–3], [4,8,–7]} spans $\mathbb{R}^3$ or not

5. (a) Let $T: V \to W$ be a linear transformation from a vector space V to a vector space W. Define kernel of T and prove that kernel of T is a subspace of V.

(b) Determine which of the following functions are linear transformations:

(i) $T:\mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T([x_1,x_2,x_3])=[x_1+1, x_2-2, x_3]$$

(ii) $f: M_{m\times n}(\mathbb{R}) \to M_{n\times m}(\mathbb{R})$ given by $f(A)=A^T$ for all $A \in M_{m\times n}(\mathbb{R})$.

(c) Find the matrix for the linear transformation $L: p_3 \to M_{2\times 2}(\mathbb{R})$ given by