(b) State and prove Max Flow-Min Cut theorem.

(c) Five ladies have men friends as shown in the following table.

Lady	Men Friends
Karen	Stuart, Paul
Mary	Philip, Stuart
Aurie	David, Paul, Roger, Stuart
Pamela	Philip, Paul
Lynne	Stuart, Philip, Paul

Draw a bipartite graph depicting this situation. Find a way in which each lady can marry a man she knows, or use Hall's Marriage Theorem to explain why no such matching exists.

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This question paper contains 8 printed pages]

Your Roll No. : .....

Sl. No. of Q. Paper : **2636 I**

Unique Paper Code : 2352202402

Name of the Paper : Introduction to Graph Theory

Name of the Course : **Bachelor of Arts (Programme) with Mathematics as Major (DSC-2)**

Semester : IV

Time : 3 Hours ***Maximum Marks : 90***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt all the given **six** questions, selecting any **two** parts out of the three **parts (a), (b)** and **(c)** for each of the six questions.

(c) **All** the questions carry equal marks.

(d) Use of Calculator is not allowed.

1. (a) Draw the following graphs and determine how many edges each has:

(i) K_5

(ii) $K_{2,4}$

(iii) $K_{2,5}$

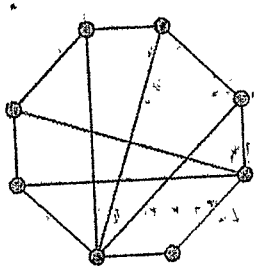
(b) Given the following degree sequences, either construct a graph with such a degree sequence, or explain why this would be impossible.

(i) 1, 1, 1, 1, 1, 1

(ii) 5, 4, 3, 2, 1

(iii) 6, 6, 4, 2, 2, 2, 2, 1, 1.

(c) Explain the difference between an Eulerian circuit and a Hamiltonian cycle. Is the following graph Eulerian? Does there exist an Eulerian trail?

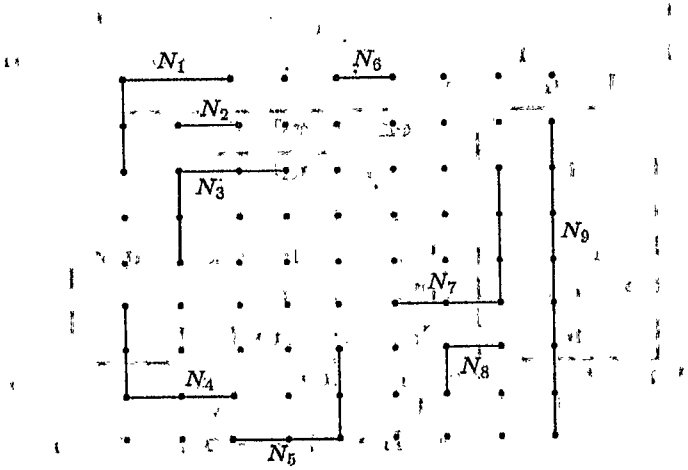


2. (a) Draw the pseudograph corresponding to each adjacency matrix:

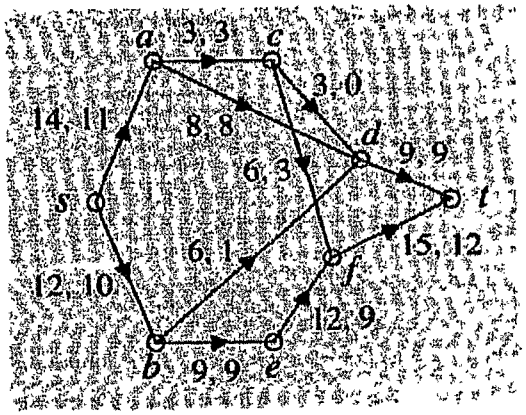
(i)
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} \end{array}$$

(ii)
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(b) Show that every graph is isomorphic to a subgraph of some complete graph. Prove that if two graphs are isomorphic, they must contain the same number of triangles.



6. (a) Define capacity of the (s,t) –cut. Find the capacity of the (s,t)–cut defined by S = {s, a, b, d} and T = {c, e, f, t} for the given network.



P.T.O.

(c) It's the weekend and Harry has several ways to spend his Friday and Saturday evenings. He can study, read, watch television or go out with the boys. Draw a tree that shows all possible ways Harry can spend his Friday and Saturday evenings.

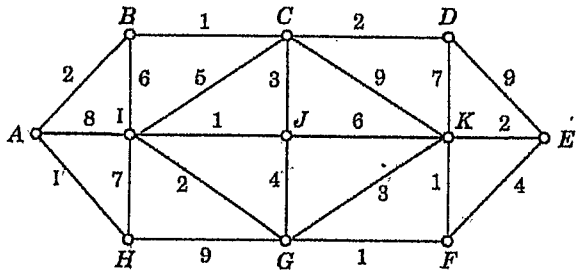
5. (a) Define a planar graph and a plane graph. Show that the complete bipartite graph $K_{2,5}$ is planar by drawing an isomorphic plane graph with straight edges. Also, label the region defined by your plane graph and list the edges that form the boundary of each region.

(b) There are ten students who, in the coming semester, will be taking the courses shown in the following table. How many time periods must be allowed in order for these students to take the courses they want without conflicts?

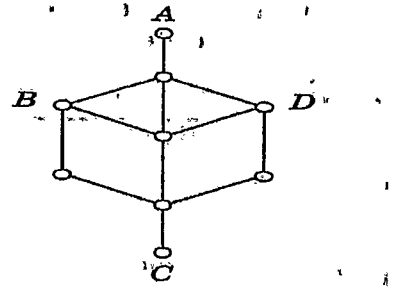
Arnold	Physics, Mathematics, English
Bill	Physics, Earth Science, Economics
Carol	Earth Science, Business
Calvin	Statistics, Economics
Eleanor	Mathematics, Business
Frederick	Physics, Earth Science
George	Business, Statistics
Huber	Mathematics, Earth Science
Ingrid	Physics, Water Skiing, Statistics
Jacquie	Physics, Economics, Water Skiing

(c) Draw the line of sight graph G associated with the given net pattern. Is this graph planar. Explain your answer.

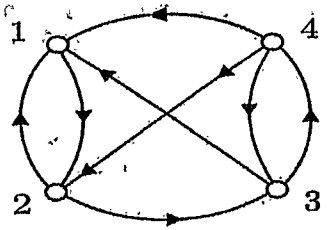
(c) Apply the first form of Dijkstra's algorithm to find a shortest path from A to E in the graph shown. Label all the vertices.



3. (a) Solve the Chinese postman problem for the following graph, where the edges of the graph represent the streets and all streets have the same unit length (starting with vertex A).



(b) Find the adjacency matrix A of the following graph and also determine the (3,3) and (1,4) entries of A^2 without calculating the matrix A^2 and explain your reasoning.



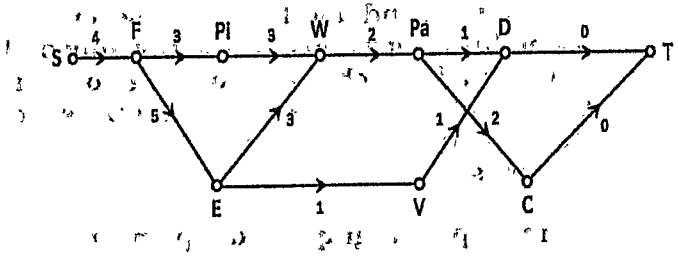
(c) Find a permutation matrix P such that

$A_2 = PA_1P^T$.

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

4. (a) To finish a basement, a contractor must arrange for certain task to be completed. As shown in below table, brief codes for reference and the time required for each activity is given.

Task	Code	Time
Floor installation	F	4
Plumbing	Pl	3
Electrical work	E	5
Wallboard	W	3
Varnish doors and moldings	V	1
Paint	Pa	2
Install doors and moldings	D	1
Lay Carpet	C	2



Referring to the above graph, find the minimum time to complete the task.

(b) Use Prim's algorithm to find the minimum spanning tree for the following graph (starting with the vertex I)