[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper	**:**	**2318**	**L**
Unique Paper Code	:	2352203602	
Name of the Paper	:	DSC- Elementary Mathematical Analysis	
Name of the Course	:	**NEP-UGCF B.A. (Prog) with Mathematics as Major**	
Semester	:	VI	

Duration : 3 Hours | Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.

1. (a) Define the limit of a function in terms of ϵ and δ. Use $\epsilon - \delta$ definition, to show that $\lim_{x\to 2}(4x-5) = 3$. (7.5)

   (b) State the sequential criterion for limits. Use it to prove that $\lim_{x\to x_0} f(x)$ does not exist, where $f(x) = \begin{cases} 1, & if\ x\ is\ rational \\ -1, & if\ x\ is\ irrational. \end{cases}$ (7.5)

   (c) Let $f: A \to \mathbb{R}$ where $A \subseteq \mathbb{R}$ and let x_0 be a cluster point of A. Then $\lim_{x\to x_0} f(x) = L$ if and only if for each sequence $< x_n >$ in $A \setminus < x_0 >$ that converges to x_0, the sequence $< f(x_n) >$ converges to L. (7.5)

2. (a) Define the continuity of a function at a point in terms of $\epsilon - \delta$. Use $\epsilon - \delta$ definition, to show that $f(x) = 4x^2 - 5x - 3$ is continuous at $x_0 = 2$. (7.5)

   (b) State the Intermediate Value Theorem. Also, prove that $\cos x = x$ for some x in $\left(0, \frac{\pi}{2}\right)$. (7.5)

   (c) Define uniform continuity of a function f on interval I. Use it to prove that the function $f(x) = 3x^2 - 2x - 1$ is uniformly continuous on the interval [-1, 5]. (7.5)

3. If $f: [a, b] \to \mathbb{R}$ is a bounded function and $\mathcal{P}$ and $\mathcal{Q}$ are any partitions of $[a, b]$, then prove that $\underline{S}(f, \mathcal{P}) \leq \bar{S}(f, \mathcal{Q})$.

P.T.O.

(b) Prove that $f(x) = 7$ is integrable on $[-1, 1]$. Also, find $\int_{-1}^{1} f$. (7.5)

(c) Let $f(x) = 5 - 3x$ be defined on $[0, 2]$. For the partition $\mathcal{P} = \{0, \frac{1}{2}, 1, \frac{3}{2}, 2\}$ of $[0, 2]$, find $\underline{S}(f, \mathcal{P})$ and $\bar{S}(f, \mathcal{P})$. (7.5)

4. (a) Use sequential criteria for integrability to show that the function $f(x) = x^2$ is integrable on $[0, 1]$ and find $\int_0^1 f$. (7.5)

(b) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = \begin{cases} 1, & if\ 1 \le x \le 3 \\ 0, & otherwise \end{cases}$. Then prove that f is integrable on $[0, 5]$ and find $\int_0^5 f$. (7.5)

(c) Let $f, g : [a, b] \to \mathbb{R}$ be integrable functions on $[a, b]$.

(i) If $f(x) \ge 0$, for all $x \in [a, b]$, then prove that $0 \le \int_a^b f$.

(ii) If $f(x) \le g(x)$, for all $x \in [a, b]$, then prove that $\int_a^b f \le \int_a^b g$. (7.5)

5. (a) Discuss the pointwise convergence of the following sequences of functions:

$$(i)\ < f_n > \ where\ f_n(x) = \frac{\sin(nx + n)}{n}, \ x \in \mathbb{R}$$

$$(ii) < g_n > \ where\ g_n(x) = \frac{x^2 + nx}{n}, \ x \in [0,1].$$ (7.5)

(b) Define uniform convergence for a sequence of functions.

Let $f_n(x) = x^n$, $x \in [0, 1]$, then show that $< f_n >$ is uniformly convergent on $[0, c]$ $where\ c < 1$ but it is not uniformly convergent on $[0, 1]$. (7.5)

(c) State the Cauchy Criterion for uniform convergence of a sequence of functions. Test the sequence $< f_n >$ for uniform convergence on $[0, 1]$ where

$$f_n(x) = \frac{x}{n}, \qquad \forall n \in \mathbb{N}, x \in [0, 1]$$

6. (a) State Weierstrass M-Test for uniform convergence of series of functions. Test the following series for uniform convergence on $\mathbb{R}$, (7.5)

$$(i) \sum_{k=1}^{\infty} \frac{\sin kx}{k^2}, \qquad (ii) \sum_{k=1}^{\infty} r^k \cos kx,\ 0 < r < 1.$$

(b) Find the interval of convergence and the radius of convergence for the following power series, (7.5)

$(i)\ \sum_{k=1}^{\infty} \frac{x^k}{k^k}$ $\qquad (ii)\ \sum_{k=1}^{\infty} \left(\frac{k+1}{k}\right)^k (x + 4)^k$

(c) Show that $tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$, $-1 \le x \le 1$.

And use Abel's theorem to deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ (7.5)

(700)