(This question paper contains 3 printed pages)

Your Roll No…………..…..

Sr. Number of the Question paper : 1774

Unique Paper Code : 2222013603

Name of the Paper : Statistical Analysis in Physics

Name of the Course : B.Sc.(H) Physics

Semester : VI (DSC)

Duration : 2 Hours Maximum Marks : 60

Instructions for Candidates

Write your Roll No. on the top immediately on receipt of this question paper.
Attempt FOUR questions in all.
Question number 1 is compulsory.
Non-programmable calculators are allowed.
Use of statistical tables are allowed.

1. Attempt any five questions from the following. 5x3

(a) What do you understand by the term Probability Density Function (PDF)? State at least three properties of a PDF. How can the probability of an interval be obtained from a given PDF?

(b) A medical survey shows that 8% of a population suffers from a certain disease. A diagnostic test gives a positive result with probability 0.92 when the disease is present and gives a false positive result with probability 0.04 when the disease is absent. Find the probability that a person actually has the disease given that the test result is positive.

(c) State the Central Limit Theorem. Discuss the importance of this theorem important in statistics and machine learning.

(d) Two fair coins are tossed simultaneously. Let (X) denote the number of heads obtained on the first coin toss and (Y) denote the total number of heads obtained in both tosses. Find the covariance between (X) and (Y).

(e) If a joint probability density function is denoted by $f_{XY}(x, y)$, give the formula to derive marginal distribution function over the variable X and Y, when the range for continuously varying parameters x, y are given by $(-\infty \leq x, y \leq \infty)$

(f) What is meant by prior distribution in Bayesian statistics? If the likelihood follows a Poisson distribution, identify the conjugate prior and write its expression.

2.

(a) A discrete random variable X follows a Binomial distribution with parameters n and p. If its probability mass function is given by

$$\binom{n}{r} p^r (1-p)^{n-r} \quad where\ r = 0, 1, 2, \ldots \ldots .. n\ and\ 0 < p < 1$$

Using this representation, derive $E(X)$ and $Var(X)$

(b) What is meant by marginal probability distribution? Let X and Y be jointly continuous random variables having joint density function

$$f_{XY}(x, y) = Ax^2 + y, \qquad for\ 0 \le x \le 1, 0 \le y \le 1$$

$$= 0 \qquad otherwise$$

Find the value of A and $P(0 \le X \le \frac{1}{2}, 0 \le Y \le \frac{1}{2})$

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3.

(a) A distribution consists of three components with frequencies of 200, 250 and 300, having means of 25, 10 and 15, and standard deviations of 3, 4 and 5 respectively. Determine the mean and standard deviation of the combined distribution.

(b) Let two random events occurring together be recorded as X = 1, 5, 3, 8, 9, 6, 5, 3 and Y = 14, 11, 17, 19, 15, 11, 19, 14. Determine the covariance matrix, correlation matrix in this case and carry out the eigen-decomposition.

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4.

(a) State and derive Bayes' theorem for continuous random variables. Explain the role of prior and likelihood in Bayesian inference.

(b) In 1989, there were three candidates for the position of principal — Mr. Chatterji, Mr. Iyengar, and Dr. Singh – whose chances of getting the appointment were in the ratio 4:2:3, respectively. The probability that Mr. Chatterji, if selected, would introduce co-education in the college is 0.3. The corresponding probabilities for Mr. Iyengar and Dr. Singh are 0.5 and 0.8, respectively. If the college introduced co-education in 1990, find the probabilities that it was introduced by:

i) Mr. Chatterji
ii) Mr. Iyengar
iii) Dr. Singh

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5.

(a) What do you understand by posterior odds? At a railway crossing, trains pass at an average rate of 4 per hour.

1. Find the probability that no train passes in 30 minutes.
2. Find the expected number of trains passing in 3 hours.

(b) A crime investigation reveals fingerprints of two individuals at the crime scene. The probability of a random match for a fingerprint pattern is (0.002). A suspect (T) matches one of the fingerprints found. Discuss using Bayes' theorem whether this evidence strongly supports the hypothesis that (T) was present at the crime scene.

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6.

(a) Explain Bayesian parameter estimation. Discuss the importance of posterior distribution in Bayesian inference.

(b) In a Bayesian linear regression, the prior (θ) follows the normal distribution with mean 1 and standard deviation 1. If the data is as follows: (Assume variance = 1)

X = [1,2,3,4]
Y = [2,5,7,9]
Compute the posterior distribution of the θ.

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