[This question paper contains 2 printed pages.]

Your Roll No...............

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Sr. No. of Question Paper : 2404

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : **B.A. / B.Sc. (Programme) with Mathematics as Non-Program : Major/Minor – DSC**

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. The use of a calculator is not allowed.

1. (a) Define a group and show that the set of integers under multiplication is not a group. (7.5)

   (b) Show that $G = \left\{\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \neq 0, a \in R\right\}$ is an abelian group under multiplication. (7.5)

   (c) Find the inverse of $\begin{bmatrix} 7 & 4 \\ 1 & 5 \end{bmatrix} \in GL(2, Z_{13})$. (7.5)

2. (a) Define the order of an element "a" in G. Also find the orders of every element of $U(15)$. (7.5)

   (b) Define center of a group and centralizer of $a \in G$. Prove that $C(a) = C(a^{-1})$. (7.5)

   (c) Prove that in any group an element and its inverse will have the same order. (7.5)

3. (a) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$. Write α, β and $\alpha\beta$ as products of disjoint cycles and as products of 2-cycles. (7.5)

   (b) List the elements of the subgroups $\langle 3 \rangle$ and $\langle 7 \rangle$ in $U(20)$. (7.5)

   (c) Let $H = \{0, \pm n, \pm 2n, \pm 3n, ...\}$. Find all left cosets of H in Z. (7.5)

4. (a) Let H be a normal subgroup of a group G and K be any subgroup of G, then prove that $HK = \{hk \mid h \in H, k \in K\}$ is a subgroup of G. (7.5)

(b) Write down a complete Cayley table for D_3, the group of symmetries of a triangle. Is D_3 Abelian? (7.5)

(c) Let $\boldsymbol{R}^*$ be a group of nonzero real numbers under multiplication. Show that $\phi = A \to \det A$ is a homomorphism from $GL(2, \boldsymbol{R})$ to $\boldsymbol{R}^*$. What is the Kernel. (7.5)

5\. (a) State the subring test. Prove or disprove that the set (7.5)

$$R = \left\{ \begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix} \mid a, b \in \mathbb{Z} \right\}$$

is a subring of the ring of all 2×2 matrices over $\mathbb{Z}$.

(b) Define characteristic of a ring. Let R be a ring with unity 1. Prove that if 1 has infinite order under addition then $charR = 0$. If 1 has order n under addition, then $charR = n$. (7.5)

(c) Prove that a finite Integral Domain is a field. Hence, show that $\mathbb{Z}_p$ is a field, where p is a prime number. (7.5)

6\. (a) Let $f: R \to S$ be a ring homomorphism. Prove that (7.5)

(i) $Kerf$ is an ideal of R.

Every ideal A of a ring R is the kernel of a ring homomorphism of R.

(b) Prove that if an ideal I of a ring R contains a unit, then $I = R$. Hence, prove that the only ideals of a field F are $\{0\}$ and F itself. (7.5)

(c) State and prove the first Isomorphism Theorem for Rings. (7.5)