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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **221**

Unique Paper Code : **2362572401**

Name of the Paper : **Optimization Techniques**

Name of the Course : **B.Sc. (Programme) Mathematical Science/ B.A. (Programme)**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *five* questions.

All questions carry equal marks.

Use of non-programmable calculator is allowed.

1. (*a*) Discuss the first and second order necessary conditions for finding extrema in single-variable optimization problem. 8

(*b*) Consider the problem of minimization of $f(x_1, x_2) = 4x_1^2 + 6x_2^2 - 8x_1x_2$ over $(x_1, x_2) \in R^2$. Solve this given problem by Newton's method starting with $x^{(0)} = (-5, 10)$. 10

P.T.O.

2. (*a*) Determine the local maximum or minimum point of the given function : 10

$$f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + x_3^2 + x_1x_2 - 2x_3 - 7x_1 + 12.$$

(*b*) Define quadratic form. Test the definiteness of the following function : 8

$$f(X) = 2x_1^2 + 2x_2^2 - 5x_3^2 - 6x_1x_2 + 2x_1x_3 + 6x_2x_3.$$

3. (*a*) Explain the following terms : 8

(*i*) Unconstrained optimization with example

(*ii*) Hessian matrix.

(*b*) Define Convex and Concave functions. Check whether the following functions are convex, concave or neither : 10

(*i*) $f(X) = x_1^2 + x_2^2 + x_3^2 - 2x_1x_2$

(*ii*) $f(X) = x^4 + 2x^2 - 2x.$

4. (*a*) Why are all goal linear programming problems minimization problems ? Can a goal programming problem be infeasible ? Discuss. 8

(*b*) Formulate the following goal programming problem and solve it graphically : 10

A Firm produces two products A and B. Each product must be processed through 2 departments. Department 1 has 30 hours of production capacity

per day and Department 2 has 60 hours of production capacity per day. Each unit of product A requires 2 hours in department 1 and 6 hours in department 2. Each unit of product B requires 3 hours in department 1 and 4 hours in department 2. Management has ranked the following goals in order of priority.

It would like to achieve the following goals in its daily product mix :

P1 : Minimize the under achievement of joint total production of 10 units.

P2 : Minimize the under achievement of producing 7 units of product B.

P3 : Minimize the under achievement of producing 8 units of product A.

5. Solve the following goal programming problem using modified simplex method: 18

Min $$Z = p_1 d_1^- + p_2 d_2^- + p_3 d_3^+ + p_4 d_4^-$$

Subject to $$7x_1 + 6x_2 + d_1^- - d_1^+ = 30$$

$$2x_1 + 3x_2 + d_2^- - d_2^+ = 12$$

$$6x_1 + 5x_2 + d_3^- - d_3^+ = 30$$

$$x_2 + d_4^- - d_4^+ = 7$$

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+ >= 0.$$

P.T.O.

6. (*a*) Explain the Pure, Mixed and 0-1 integer programming problem. Discuss their formulation with examples. 8

(*b*) The Live wright Medical Supplies Company has a total of 12 salespeople it wants to assign to three regions—the South, the East, and the Midwest. A salesperson in the South earns \$600 in profit per month for the company, a salesperson in the East earns \$540, and a salesperson in the Midwest earns \$375. The southern region can have a maximum assignment of 5 salespeople. The company has a total of \$750 per day available for expenses for all 12 salespeople. A salesperson in the South has average expenses of \$80 per day, a salesperson in the East has average expenses of \$70 per day, and a salesperson in the Midwest has average daily expenses of \$50. The company wants to determine the number of salespeople to assign to each region to maximize profit. Formulate an integer programming model for the problem. 10

7. (*a*) Discuss the real-life applications of integer linear programming problem. 8

(*b*) Solve the following mixed integer program by the branch and bound algorithm : 10

$$\text{Minimize : } Z = 10x_1 + 9x_2$$

$$\text{Subject to : } x_1 \leq 8$$

$$x_2 \leq 10$$

$$7x_1 + 4x_2 \geq 45$$

$x_1 \geq 0$, $x_2 \geq 0$ and x_2 is an integer.