[This question paper contains 4 printed pages.]

**Your Roll No...............**

**Sr. No. of Question Paper : 1817** **L**

Unique Paper Code : 2372012401

Name of the Paper : DSC: Sampling Distributions (NEP)

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all by selecting **three** questions from each section.
3. **All** questions carry equal marks.
4. Use of simple calculator is allowed.

**SECTION I**

1. (a) State Chebychev's Inequality. If $X$ has the distribution with p.d.f. $f(x) = e^{-x}, x \geq 0$, use Chebychev's inequality to obtain a lower bound to probability of the inequality $-1 \leq X \leq 3$, and compare it with the actual probability.

   (b) If $X_i$ can have only two values with equal probabilities $i^{\alpha}$ and - $i^{\alpha}$, show that WLLN can be applied to the independent variables $X_1, X_2, \ldots$, if $\alpha < \frac{1}{2}$.

   [8,7]

2. (a) If $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with mean $\mu$ and variance $\sigma^2$ (finite). Then prove that for $-\infty < a < b < \infty$, $\lim_{n \to \infty} P_r[a \leq \frac{S_n - n\mu}{\sigma\sqrt{n}} \leq b] = \int_a^b \frac{1}{\sqrt{2\pi}} e^{-(\frac{x^2}{2})} dx$

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a population with p.d.f. $f(x) = 1, 0 < x < 1$. Show that $Y_1 = \frac{X_{(1)}}{X_{(2)}}, Y_2 = \frac{X_{(2)}}{X_{(3)}}, \ldots, Y_{n-1} = \frac{X_{(n-1)}}{X_{(n)}}$ and $Y_n = X_{(n)}$ are independently distributed and identify their distributions.

[7, 8]

3. (a) Define convergence in law, convergence in probability and convergence with probability one. State the relationship between convergence in probability and convergence with probability one and prove it.

(b) For the exponential distribution, $f(x) = e^{-x}, x \geq 0$, find the p.d.f. of the sample range $W$ in a random sample of size n and show that $E[W] = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n-1}$.

[8,7]

4. (a) What is meant by a statistical hypothesis? What are the two types of errors of decision that arise in testing a hypothesis? Briefly explain how a statistical hypothesis is tested.

(b) Explain the following terms:
   (i) P-value and level of significance
   (ii) Critical region and significant value
   (iii) Sampling distribution of a statistic
   (iv) Test statistic for the test of significance for difference of proportions

[7,8]

## SECTION II

5. (a) Find the mean deviation about mean for t-distribution with n d.f.

(b) If $F \sim F_{m,n}$ and $\lim n \to \infty$ then prove that $mF$ follows chi-square distribution with $m$ d.f..

(c) What is contingency table? Describe how the chi-square distribution may be used to test whether the two attributes in an $m \times n$ contingency table are independent.

[5, 5, 5]

6. (a) If $X$ and $Y$ are independent chi-square variates with $m$ and $n$ d.f. respectively, Show that $U = X + Y$ and $V = \frac{nX}{mY}$ are independently distributed. Also, find the distribution of $V$.

(b) It is required to test that the mean of a normal population is zero against it is greater than zero. Show that the t-test for acceptance of hypothesis based on a random sample of size n reduces to $\left(\sum_{i=1}^{n} x_i\right)^2 \le \frac{nt_\alpha^2}{t_\alpha^2 + (n-1)}\left(\sum_{i=1}^{n} x_i^2\right)$, where $t_\alpha$ is the value of Student's t at the desired level of significance $\alpha$ for (n-1) degrees of freedom.

[8, 7]

7. (a) $X$ is a Poisson variate with parameter $\lambda$ and $\chi^2$ is a chi-square variate with $2k$ d.f.. Prove that for all positive integers $k$, $P_r[X \le k-1] = P_r[\chi^2 > 2\lambda]$

(b) Let $X_1$ and $X_2$ are independently distributed as $\chi^2$-variate with 2 d.f.. Find the distribution of $Z = \frac{1}{2}(X_1 - X_2)$.

[8, 7]

8. (a) Suppose $X_1, X_2, \ldots, X_n$ $(n > 1)$ are independent variates each distributed as $N(0, \sigma^2)$. Find the p.d.f. of $W = \frac{X_1}{\sqrt{\frac{1}{n}\sum_{i=1}^{n} X_i^2}}$. Why does not $W$ follow the t-distribution?

(b) Prove that if $n_1 = n_2$, the median of F-distribution is at F = 1 and that the quartiles $Q_1$ and $Q_3$ satisfy the condition $Q_1 Q_3 = 1$.

[8,7]