(i) $f_r(x) = rx^2(1 - x)$

(ii) $f_c(x) = -c(x^2 - 3x + 2)$

(c) Convert $I_{n+1} = I_n - rI_n + sI_n(1 - \frac{I_n^2}{N^2})$ into a one-parameter family.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 5784 **L**

Unique Paper Code : 2354000011

Name of the Paper : Discrete Dynamical Systems

Name of the Course : **Common Program Group**

Semester : VI

Duration : 3 Hours Maximum Marks : 90

<u>Instructions for Candidates</u>

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are campulsory and attempt any **two** parts of each question.
3. Each question carries equal marks.
4. Non-programmable scientific calculator is allowed.

1. (a) Suppose a population, initially 836,900, has a birth rate of 9% and a death rate of 7.5% per generation. Construct the model that satisfies:

(i) Immigration occurs at 6800 per generation.

(ii) Migration occurs at 4340 per generation.

(b) (i) The intrinsic growth rate of the first species is 1.6 and that of the second is 1.3; each species' growth is reduced by 0.35 times the population of the other species.

(ii) In addition, the first species receives a constant immigration of 3000 individuals per time step, while the second species experiences emigration of 1500 individuals per time step.

(c) Construct a model that satisfies :

(i) In a population of size 250,000, recovery is not possible, and if 1200 are ill, that number will double in the following week.

(b) (i) In $x_{n+1} = x_n^3 + 2x_n$, find all fixed points and use the derivative to determine their stability and any oscillation around them.

(ii) In characterizations of chaos define dense orbit.

(c) Find a 2-cycle of $x_{n+1} = 1 - x_n^2$ and determine its stability.

6. (a) Define the intervals of existence and stability. For the price model

$p_{n+1} = \frac{1}{p_n} + p_n + c$, find the positive equilibrium point and its intervals of existence and stability.

(b) In some kind of bifurcation two fixed points, one stable and another unstable come into existence simultaneously at the same location and for the same parameter value. Show that this occurs for each of functions given below :

of parents in that region includes 35% who are college educated and 65% who are not, what will that distribution be three generations from now?

(b) For the prey-predator model with constant immigration $P_{n+1} = 0.8P_n - 0.3Q_n + 8000$; $Q_{n+1} = 0.2P_n + 0.9Q_n + 2000$. Find the fixed point and determine whether it is a sink, source or saddle why iterating and graphing in solution space the first few iterates for several choices of initial conditions.

(c) Find the equilibrium population levels, if any, for Nonlinear Population model

$P_{n+1} = 3P_n e^{-P_n/5000}$. Also discuss their stability.

5. (a) What is a Cobweb graph? Draw a cobweb graph for $x_{n+1} = x_n^2$ starting with $x_0 = 1.1$.

(ii) Each week 45% of those ill in a population of size 359,700 are still sick, and the maximum number of new cases possible in any week is 70,000.

2. (a) Construct and Explain Linear Price-supply Model. Construct a linear price-supply model by assuming instead that demand is either constant or reacts immediately to price :

$$D_n = k_1 - c_1P_n$$

(b) The following are overlapping-generations models that have been converted into systems. Find the original equation for :

(i) $P_{n+1} = 1.6\,P_n + 0.4\,Q_n$, $Q_{n+1} = P_n$

(ii) $P_{n+1} = 0.9\,P_n + Q_n$, $Q_{n+1} = P_n + 500$

(c) Construct the linear competition model with no immigration, migration or harvesting that satisfies :

(i) $r_1 = 2.5$, $r_2 = \frac{4}{3}$, $P_0 = 1200$, $Q_0 = 1400$, $P_1 = 1300$, $Q_1 = 850$

(ii) $P_0 = 150$, $Q_0 = 250$, $P_1 = 180$, $Q_1 = 120$, $P_2 = 300$, $Q_2 = 50$

3. (a) When a population has a birth rate b that its death rate d, then that population will decrease over time and may eventually become extinct. For a population with b=0.15, d=0.25 and initial size of 800 :

(i) Find the equation for the population size P_n after n generations;

(ii) Determine how long it will take for the population to fall below 1, which we interpret as extinction.

(b) Suppose that a savings account pays 5% annual interest compounded monthly. If \$6000 is initially deposited :

(i) How much will be in the account after 4 years?

(ii) When will the account balance reach \$9500?

(c) Suppose that every hour the number of cells of a certain virus doubles. How long will it take for the number of cells to increase by 10,000 if initially there are :

(i) 2000 cells?

(ii) 20,000 cells?

4. (a) Suppose that in a certain part of the country 90% of the children of college educated parents get a college education, but 10% do not. Also, among parents who are not college educated, 40% of their children get a college education, but the other 60% do not. Assuming that the current distribution