

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5526

L

Unique Paper Code : 2374000023

Name of the Paper : Statistics in Finance (NEP)

Name of the Course : GE

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt a total of **six** questions in all.
3. Use of a non-programmable calculator is permitted.

Q1 Attempt any three parts. All parts carry equal marks.

- i. Define nominal and effective rates of interest. Explain the relationship between them and explain the impact of compounding frequency. Find the corresponding effective rates for nominal rates: 6% compounded monthly and 12% compounded quarterly.
- ii. If you receive 6% interest compounded yearly, approximately how many years will it take for your money to quadruple? What is it if this interest was only 3% ?
- iii. \$1000 is kept in a savings account at 3% compounded continuously for 3 years. It is then withdrawn and placed in another bank account at a rate of 4% compounded continuously for 5 years. What will be the balance in the second account after 5 years?
- iv. An investor sold 2000 calls and 1500 puts with the same exercise price and same maturity on the same underlying stock. Delta for the call is 0.42, and delta for the put is -0.38. How many stocks should the investor sell or buy in order to have a delta-neutral position?
(5X3=15marks)

Q2 (a) An investor has a capital of ₹ 8,400 at his disposal to buy stocks whose current price is ₹100. Further put options on the same stock with a delivery price of $K = ₹100$ and a time to maturity of 1 year are quoted at a market price of ₹5 per contract. Consider two alternative investment plans:

P.T.O.

Portfolio A: Buying 84 stocks

Portfolio B: Buying 80 stocks for 8,000 and buying 80 put options for 400.

Discuss the effect of portfolio insurance on portfolio value and return.

- (b) Explain 'The Comparison Principle' with an example in the context of financial markets. Consider the 5-year coupon bonds: one has a coupon of 9% and sells for Rs. 101.00, and the other has a coupon of 7% and sells for Rs. 93.20. Find the price of a 5-year zero-coupon bond. (8,7)

Q-3 (a) What do you mean by Derivatives? Explain the types of derivatives with examples.

(b) Draw the payoff matrix for the following strategies:

(i) Consider the following strategy where $K=100$

- The purchase of one stock
- The purchase of one European call with exercise price K
- The sale of one European call with exercise price K

(ii) Consider the following option combination strategy,

- 2 Long calls with strike price $K_0 = 45$ USD and price $C = 10$ USD.
- A Long put with strike price $K_0 = 45$ USD and price $P = 12$ USD.

(7,8)

Q4 (a) Explain a yield curve. What do you mean by discount factors, and how do you calculate the present value by using the discount factor? Also, give the formula for the spot price for a 2-year bond.

- (b) Suppose that 6-month, 12-month, 18-month, 24-month, and 30-month zero rates are, respectively, 4%, 4.2%, 4.4%, 4.6%, and 4.8% per annum, with continuous compounding. Estimate the cash price of a bond with a face value of 100 that will mature in 30 months and pay a coupon of 4% per annum semiannually under continuous compounding. (8, 7)

Q5 (a) The price of a stock is \$50 at time $t=0$. It is estimated that the price will be either \$25 or \$100 at $t=1$. It is assumed that no dividends are paid. A European call with an exercise price of \$50 is available at $t=0$. This call will expire at $t=1$. The market rate of interest is 25% per annum compounded annually.

i. What return can the owner of the following portfolio have at $t=1$:

- Sell 3 calls for price C each
- Buy 2 stocks for \$50 each
- Borrow \$50 at market rate of interest (25% per annum)

ii. Calculate the price C of the call

- (b) Define the future and forward contract. Consider a long forward contract on a 5-year bond, which is currently traded at a price of 900 EUR. The delivery price is 910 EUR, and the time to maturity of the forward contract is one year. The bond pays a coupon of 60 EUR after 6th and 12th months. The estimated compounded annual interest rates for 6th and 12th months are 9% and 10%, respectively. Determine the price of the contract. (8,7)

- Q6 (a) Consider a long forward contract to buy an object that has a price of S_t at time t . Assume K to be the delivery price at the time T of maturity. The value of the long forward at time t is $V(S, \tau)$; where τ is the time to maturity. If the underlying object does not pay any dividends and does not involve any costs during the time to maturity τ , then show that the value of a long forward at time t is obtained as:

$$V(S_t, \tau) = V_{K,T}(S_t, \tau) = S_t - K e^{-r\tau}$$

where r is the rate of interest. Further prove that the forward price is $F_t = S_t e^{r\tau}$.

- (b) Define 'intrinsic value' of a call and put option at time t . Under what conditions is an option said to be 'in-the-money', 'at-the-money' and 'out-of-money'. (8,7)

- Q7 (a) The current price of a certain stock is \$100. During each of the following two periods, the stock prices will rise by 1/10 or fall by 1/11. The risk-free rate of interest is 5% each period. Calculate the price of a European put option with time to expiration of 2 periods and exercise price = $K = \$102$. Using the binary model, determine the price of puts.

- (b) Explain the Binomial Process for a stochastic process in discrete time. Also show that for the large t , the Binomial process X_t follows a normal distribution with mean zero and variance $t (\Delta x)^2$. (7,8)

- Q8 (a) Consider the following collar portfolio which consists of

- One share with $S_0 = 25$
- One long put with $K_1 = 22.5$, $\tau = 1$ year
- One short call with $K_2 = 27.5$, $\tau = 1$ year

Using Black-Scholes' model, it has been estimated that corresponding to

$$K_1 : \phi(d_1) = 0.53 \quad \text{and} \quad K_2 : \phi(d_2) = 0.22$$

- i. Draw the payoff graph of the caller at the option expiration date.
- ii. If the stock price goes up by 1, what will be the gain or loss from the portfolio?
- iii. What happens to the portfolio if the stock prices become very high or very low?

(b) Obtain the Greek 'theta' of a European put option.

(9,6)