[This question paper contains 2 printed pages.]

Your Roll No...............

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Sr. No. of Question Paper : **5417**

Unique Paper Code : 2354000017

Name of the Paper : Elements of Partial Differential Equations

Name of the Course : **Common Prog Group**

Semester : VIII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question. Each part carries **7.5** marks.
4. Use of Calculator not allowed.

1.(a) (i) Eliminate the arbitrary function f from the equation

$$z = f(x^2 + y^2 + z^2, z^2 - 2xy).$$ 3.5

(ii) Eliminate the constants a and b from the equation

$$2z = (ax + y)^2 + b.$$ 4

(b) Show that how to solve, by Jacobi's method, a partial differential equation of the type

$$f\left(x, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial z}\right) = g\left(y, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$$

and illustrate the method by finding a complete integral of the equation

$$2x^2y\left(\frac{\partial u}{\partial x}\right)^2 \frac{\partial u}{\partial z} = x^2 \frac{\partial u}{\partial y} + 2y\left(\frac{\partial u}{\partial x}\right)^2.$$ 7.5

(c) (i) Find the general solution of the PDE $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z.$ 4.5

(ii) Find the general integral of the partial differential equation $pq = 1.$ 3

2.(a) Find the general integral of the partial differential equation

$$(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$$

through the curve $xz = a^3, y = 0.$ 7.5

(b) Find the equation of the system of surfaces which cut orthogonally the cones of the system $x^2 + y^2 + z^2 = cxy.$ 7.5

(c) Show that the equation $z = px + qy$ is compatible with any equation $f(x, y, z, p, q) = 0$ that is homogeneous in x, y and z. Solve completely the simultaneous equations $z = px + qy, 2xy(p^2 + q^2) = z(yp + xq).$ 7.5

3. (a) Solve $(2D^2 - 5DD' + 2D'^2)z = 24(y - x).$ 7.5

(b) Find the particular integral of the partial differential equation

$$(D^2 - 2DD' + D'^2)z = e^{x+2y}.$$ 7.5

(c) Find the general solution of the partial differential equation

$$(D - 1)(D - D' + 1)z = Cox(x + 2y).$$ 7.5

4. (a) Solve $(x^2D^2 - y^2D'^2)z = xy\,.$ 7.5

(b) Solve by Monge's Method $x^2r + 2xys + y^2t=0.$ 7.5

(c) Solve using Monge's Method $pt - qs = q^3.$ 7.5

P.T.O.

5. (a) Using separation of variables, show that the one-dimensional wave equation

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{16}\frac{\partial^2 z}{\partial t^2}, \quad 0 < x < a, t > 0$$
$$z(0,t) = 0, z(a,t) = 0 \text{ for all } t$$

has solution

$$z(x,t) = \sum_r \left\{A_r cos\frac{r\pi 4t}{a} + B_r sin\frac{r\pi 4t}{a}\right\} sin\frac{r\pi x}{a}.$$

Where A_r's and B_r's are constants. 7.5

(b) Solve the one-dimensional diffusion equation

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{k}\frac{\partial \theta}{\partial t}, \qquad 0 < \theta < l, t > 0$$
$$\theta(0,t) = \theta(l,t) = 0 \qquad \text{for } t \geq 0$$
$$\theta(x,0) = f(x) \qquad 0 \leq \theta \leq l$$

7.5

(c) Find the D'Alembert solution of the Cauchy problem for one dimensional wave equation given by

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}, \quad x \in R, t > 0$$
$$y = sin\, x, \frac{\partial y}{\partial t} = cos\, x \text{ at } t = 0, x \in R$$

7.5

6.(a) Determine the solution of the initial boundary value problem

$$\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, t > 0$$
$$u(x,0) = sin\, x, \qquad 0 \leq x < \infty,$$
$$u_t(x,0) = x^2, \qquad 0 \leq x < \infty,$$
$$u(0,t) = 0, \quad t \geq 0$$

7.5

(b) Determine the solution of the vibrating string problem

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < l, \qquad t > 0$$
$$u(x,0) = \begin{cases} \dfrac{hx}{a}, & 0 \leq x \leq a \\ \dfrac{h(l-x)}{l-a}, & a \leq x \leq l \end{cases}$$
$$u_t(x,0) = 0, \qquad 0 \leq x \leq l,$$
$$u(0,t) = 0, \qquad t \geq 0$$
$$u(l,t) = 0, \qquad t \geq 0$$

7.5

(c) Derive the equation of traffic flow

$$\frac{\partial \rho}{\partial t} + c(\rho)\frac{\partial \rho}{\partial x} = 0,$$

Where $\rho(x,t)$ is traffic density, flow rate q depends only on density, i.e., $q = \rho v(\rho)$ and $\quad c(\rho) = q'(\rho) = v + \rho v'(\rho)$ 7.5