5. (a) Find the Maclaurin series for the function $f(z)=\cos(z)$.

(b) Find the Laurent series of the function

$$f(z)=\frac{1}{z(1+z^2)}, \text{ when } 0<|z|<1.$$

(c) For the given function, $f(z)=\frac{z^2+2}{z-1}$, show any singular point is a pole. Determine the order of each pole and find the corresponding residue.

6. (a) State Cauchy Residue Theorem and use it to evaluate the integral of $\frac{1}{1+z^2}$ around the circle $|z| = 2$ in the counterclockwise.

(b) Prove that

$$\text{Res}_{z=-1}=\frac{\text{Log}z}{\left(z^2+1\right)^2}=\frac{\pi+2i}{8}$$

(c) Find the value of the integral

$$\int_C \frac{dz}{z^3(z+4)},$$

taken counterclockwise around the circle $|z+2|=3$.

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This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **5418** **I**

Unique Paper Code : 2354000018

Name of the Paper : Elements of Complex Analysis-GE

Name of the Course : **COMMON PROG GROUP**

Semester : VIII

Time : 3 Hours ***Maximum Marks : 90***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **all** questions by selecting **two** parts from each question.

(c) Each question carries **15** marks each.

(d) Use of Calculator is **not** allowed.

1. (a) Let f be a function defined at all the points z in some deleted neighborhood of a point z_0 and $w_0 \in \mathbb{C}$. Define $\lim_{z\to z_0} f(z) w_0$. Further, if $f(z)=\frac{z}{\bar{z}}$, then show that $\lim_{z\to 0} f(z)$ does not exist.

(b) show that $\lim_{z\to z_0} f(z)g(z)=0$ if $\lim_{z\to z_0} f(z)=0$ and if there exists M > 0 such that $|g(z)|\le M$ for all z in some neighborhood z_0.

(c) Let f be a function defined on a domain containing a neighborhood of a point z_0. Then define the derivative of f at z_0. Further, using the definition, show that the derivative of $f(z)=z^2$ is 2z.

2. (a) Using the Cauchy-Riemann equations, show that $f(z)=|z|^2$ differentiable at z = 0 only.

(b) let $f: \mathbb{C}\to\mathbb{C}$ be a function such that f'(z) = 0 for all z in a domain D. show that f(z) must be a constant function throughout D.

(c) Show that

(i) $\exp\left(\frac{2+\pi i}{4}\right)=\sqrt{\frac{e}{2}}(1+i)$

(ii) $\mathrm{Log}(1-i)=\frac{1}{2}\ln 2-\frac{\pi}{4}i$

3. (a) if W(t) = u(t) +iv(t) is a complex–valued function of a real variable t and w'(t) exists. Show that $\frac{d}{dt}[w(-t)=-w'](-t)$, where $w'(-t)$ denotes the derivative of w(t) with respect to t, evaluated at – t.

(b) Use parametric representation for C to evaluate $\int_C \frac{Z+2}{z}dz$, where C is the semicircle $z=2e^{i\theta}$ $(\pi\le\theta\le 2\pi)$

(c) Without evaluating the integral, show that $\left|\int_C \frac{z+4}{z^3-1}dz\right|\le\frac{6\pi}{7}$, when C is the arc of the circle $|z|=2$ from z = 2 to z = 2i that lies in the first quadrant.

4. (a) By finding an antiderivative, evaluate the following integrals, where the path is any contour between the indicated limits of integration

(i) $\int_0^{\pi+2i}\cos\frac{z}{2}dz$ (ii) $\int_i^{i/2} e^{\pi z}dz$

(b) State the Cauchy-Goursat theorem and hence apply it to evaluate $\int_C f(z)dz$, where the contour C is the positivel oriented unit circle |z| = 1 and

(i) $f(z)=\frac{z^2}{e^z}$ (ii) $f(z)=\frac{z^2}{z+3}$

(c) Find the value of the integral of g(z) around the circle |z–i| = 2 in the positive sense when

(i) $g(z)=\frac{1}{z^2+4}$

(ii) $g(z)=\frac{1}{(z^2+4)^2}$