[This question paper contains 4 printed pages.]

Your Roll No................

Sr. No. of Question Paper : 5419

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Unique Paper Code : 2354000019

Name of the Paper : Optimization Techniques

Name of the Course : **Common Prog Group-NEP-UGCF 2022**

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **one** part from each question.

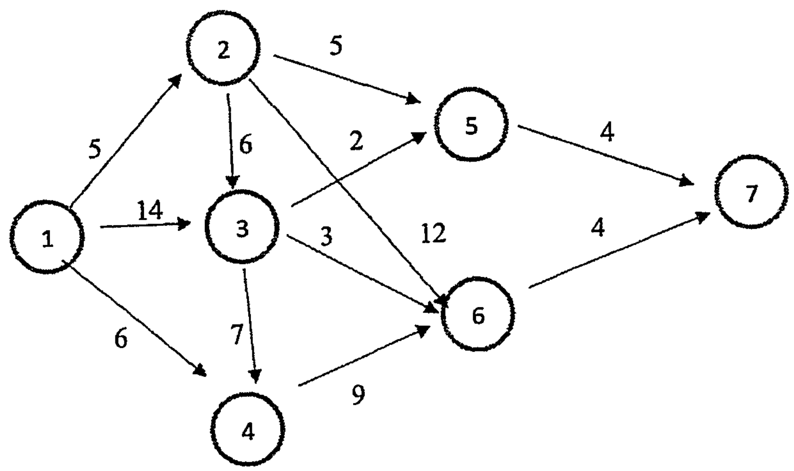
1.(a) A firm has two factories X and Y. It wishes to ship its products from its two factories to three retail stores A, B and C. The number of units available at factories X and Y are 200 and 300 respectively, while the demands at retail stores A, B and C are 100, 150 and 250 respectively. Rather than shipping directly from factories to retail stores, it is added to investigate possibility of transshipment. The unit transportation cost (in rupees) is given in the following table. Find the optimal shipping schedule.

		Source		**Destination**		
		X	**Y**	**A**	**B**	**C**
Source	**X**	0	8	7	8	9
	Y	6	0	5	4	3
Destination	**A**	7	2	0	5	1
	B	1	5	1	0	4
	C	8	9	7	8	0

(b) A 4-ton vessel can be loaded with one or more three items. The following table gives the unit weight w_i in tons and the unit revenue in thousands of dollars r_i, for item. How many units of each item be loaded to maximize the value?

Item (*i*)	**Weight (*w*)**	**Value (r)**
1	1	20
2	2	40
3	3	60

2. (a) Suppose we want to select the shortest highway route between two cities. The network in given figure provides the possible route between the starting city node '1'and the destination city at node '7'. Solve by the method of forward recursive equation and use it to find the optimal solution for the following data:



(b) In a cargo loading problem, there are 4 items of different weights/unit and different values/unit as given below:

Item (i)	Weight (w)	Value (v)
1	2	2
2	3	5
3	4	7
4	5	9

The maximum cargo load is restricted to 17. How many units of each item be loaded to maximize the value?

3. (a) Solve the following ILPP by using Gomory's Cutting Plane Method

$$\text{Max } Z = 2x_1 + x_2$$

subject to
$$x_1 + x_2 \leq 5$$
$$36x_1 + 2x_2 \leq 21$$
$$x_1, x_2 \geq 0 \text{ and both are integers.}$$

(b) Consider the following ILLP

$$\text{Max } Z = x_1 + 4x_2$$

subject to $2x_1 + 4x_2 \leq 7$

$5x_1 + 3x_2 \leq 15$

$x_1, x_2 \geq 0$ and both are integers

(i) Convert the above defined problem into the standard Linear programming problem.

(ii) Write the two LPP's by branching from x_2 and find the next branching variable, if the initial solution is $x_1 = 0$ and $x_2 = \frac{7}{4}$, the maximum value of $Z = 7$, with the last simplex table as follows:

c_j			1	4	0	0
c_B	B	x_B	x_1	x_2	s_1	s_2
4	x_2	7/4	1/2	1	1/4	0
0	s_2	39/4	7/2	0	-3/4	1
$z_j - c_j$			1	0	1	0

4. (a) Consider the following ILPP

$$\text{Max } Z = x_1 - x_2$$

subject to $x_1 + 2x_2 \leq 4$

$6x_1 + 2x_2 \leq 9$

$x_1, x_2 \geq 0$ and both are integers.

(i) Find the optimal solution of the LPP by using the simplex method.

(ii) Solve the given ILPP graphically.

(b) Consider the following optimal simplex tableau of the associated Linear Programming Problem for a given AILP (with maximization from and x_3, x_4 as slack variables) with Max $Z = 18.75$. Generate the Gomory's cut through Z and hence solve the given AILP.

c_j			5	2	0	0
c_B	B	x_B	x_1	x_2	x_3	x_4
2	x_2	1.25	0	1	0.75	- 0.5
5	x_1	3.25	1	0	- 0.25	0.5
	$z_j - c_j$		0	0	0.25	1.5

5. (a) (i) Define a convex function on a convex set S $\subseteq R^n$. Is the function $f(x) = x^2, x \in R$, convex? Justify your answer using definition.

   (ii) Suppose that S $\subseteq R^n$ be a convex set and $f: S \rightarrow R$. Prove that f is a convex function on S if and only if its epigraph E_f is a convex set.

   (b) (i) Is every convex function continuous? Explain with an example.

   (ii) Using Charnes and Cooper algorithm, solve the following linear fractional programming problem:

$$Max\ Z = \frac{x_1}{x_1+x_2+1}$$

subject to
$$x_1 + x_2 \leq 1$$
$$x_1 + 2x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

6. (a) (i) For the following problem, check whether it is a convex programming problem with justification:

$$Max\ x_2$$

subject to
$$x_1^2 + x_2^2 \leq 1$$
$$x_1^2 \geq x_2$$

   (ii) Define a quasilinear function on a convex set S $\subseteq R^n$. Give an example of a quasilinear function.

   (b) (i) Define a quasi-convex function on a convex set S $\subseteq R^n$. Is the function $f(x) = [x], x \in R$, quasi-convex? Justify your answer. Give an example of a quasi-convex function on R which is not pseudo-convex.

   (ii) Define a pseudo-concave function on a convex set S $\subseteq R^n$. Give an example of a pseudo-concave function.