Sr. No. of Question Paper : 2557 **L**

Unique Paper Code : 2353200010

Name of the Paper : Discrete Dynamical Systems

Name of the Course : **B.A. / B.Sc. Programme – DSE**

Semester : VI

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Each question contains **three** parts. Attempt **all** questions by selecting any **two** parts from each question.
3. Parts of questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of non-programmable scientific calculator is allowed.

1. (a) Determine carrying capacity for a logistic model in which (i) A population of 35,000 causes extinction in the next step. (ii) A population of $P_0 = 200,000$ produces the maximum population in the next step.

(b) Construct a disease model assuming that at each step $r\,I_n$ recover and the number of new cases is proportional to the product of

(i) $I_n^{\,2}$ and $(1 - \frac{I_n^{\,2}}{N^2})$

(ii) $I_n^{\,2}$ and $(1 - \frac{I_n}{N})$

(iii) I_n and $e^{-\frac{I_n}{N}}$

differences.

2. (a) Construct the Nonlinear Population Model II with:

   $(i)\ r = 4,\ N = 65,000$

   $(ii)\ r = 10,\ N = 10^7$

   (iii) Find the model satisfying: $N = 1000,\ P_0 = 100,\ P_1 = 250$

   (b) For the overlapping-generations model:

   $P_{n+1} = 1.2\,P_n + 0.5\,P_{n-1}$ with $P_{-1} = 1200$ and $P_0 = 1500$

   (i) Compute P_1, P_2 and P_3.

   (ii) Convert the above second-order equation into a system of first-order equations.

   (c) (i) For each of the following price models, compute $P_1, P_2 \ldots, P_5$ with the given initial price: $P_{n+1} = 8000 - 0.9\,P_n,\ P_0 = 4000$

   (ii) For the Linear Price Model $P_{n+1} = 18 - 0.8\,P_n$, find the largest value of P_0 for which $P_n \geq 0$ for all $n \geq 0$.

   (iii) For the model $P_{n+1} = 15 - 1.5\,P_n$, is there a value of P_0 for which $P_n \geq 0$ for all $n \geq 0$.

3. (a) Suppose someone borrows \$800 at an interest rate of 12% per year paid monthly. Give the iterative equation that models the unpaid balance on the loan if:

   (i) \$60 is paid back at the end of each month;

   (ii) the borrower pays back nothing at the end of the first month, \$10 at the end of the second month, \$20 at the end of the third month, \$30 at the end of the fourth month, etc.

decrease over time and may eventually become extinct. For a population with $b=0.1, d=0.3$ and initial size of 1100:

(i) Find the equation for the population size P_n after n generations;

(ii) Determine how long it will take for the population to fall below 1, which we interpret as extinction.

(c) Suppose that \$10000 is initially deposited into an account paying interest at an annual rate of 8%. How much more will be in the account after 5 years if it gets compound rather than simple interest, if compounding is done:

(i) quarterly?

(ii) monthly?

4. (a) If a population is growing by 5% per generation: (i) How many generations will it take for an initial population of 1000 to grow to 1800 ? (ii) What growth rate will make this happen in four generations?

(b) Suppose that at the end of each month someone deposits \$100 into an account earning interest at an annual rate of 6% compounded monthly.

(i) How much will this investment be worth after 10 years?

(ii) For how long would these deposits need to be made before the total value of the investment reaches \$20000?

(c) Suppose that researchers start training a group of 60 dolphins to communicate with humans. They notice that each week 20% of the untrained dolphins become trained, but 10% of the trained dolphins revert to untrained status. Assuming that originally all 60 dolphins were untrained, approximately how many will be trained and how many untrained after 6 weeks?

5. (a) Find the exact solution of $x_{n+1} = \sqrt{x_n}$ for any $x_0 \geq 0$ and use it to determine the asymptotic behavior for (a) $x_0 = 1$; (b) any $x_0 > 0$.

6. (a) Convert the disease model $I_{n+1} = I_n - rI_n + sI_n(1 - \frac{I_N}{N})$ into a one-parameter family.

(b) In some kind of bifurcation two fixed points, one stable and another unstable come into existence simultaneously at the same location and for the same parameter value. Show that this occurs for each of functions given below:

(i) $f_a(x) = a - x^2$

(ii) $f_c(x) = \frac{1}{x} + 2x - c$

(c) Show that $x_{n+1} = 1 - 2|x_n|$ has an $m - cycle$ for any integer $m \geq 1$.