[This question paper contains 2 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2360

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Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : **B.A. / B.Sc. (Programme) with Mathematics as Non-Major / Minor**

Semester : II – DSC

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **any 5** parts from Question **1** and any **2** parts from Question **2-6**.
3. Each Question carries **15** marks.
4. Use of calculator is **NOT** allowed.

1. Attempt any 5 parts: : **(All parts carry 3 marks)**

   i). Are the vectors $\boldsymbol{x} = [\mathbf{10}, -\mathbf{8}, \mathbf{3}, \mathbf{0}, \mathbf{27}]$ and $\boldsymbol{y} = \left[\frac{5}{6}, -\frac{2}{3}, \frac{3}{4}, \mathbf{0}, -\frac{5}{2}\right]$ parallel? Justify.

   ii). Is $[1, 1, 1]$ a linear combination of $[2, 0, 3]$ and $[0, 2, 1]$? Justify.

   iii). Show that for unit vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, if $(\boldsymbol{a}-\boldsymbol{b}).(\boldsymbol{a}-\boldsymbol{b}) \geq 0$, then $\boldsymbol{a}.\boldsymbol{b} \leq \mathbf{1}$.

   iv). Let $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ be vectors in $\mathbb{R}^n$. Is it true that $(\boldsymbol{x}.\boldsymbol{y}).\boldsymbol{z} = \boldsymbol{x}.(\boldsymbol{y}.\boldsymbol{z})$? Justify.

   v). Find a unit vector in the direction of vector $[2, -3, 1, 2]$.

   vi). $(-4, -1, 6), (2, 3, -2), (8, -1, 6)$ are vertices of an isosceles triangle. True or False? Justify.

   vii). Let $\boldsymbol{x}, \boldsymbol{y}$ be vectors in $\mathbb{R}^n$ and d is a scalar. Is it true that $(d\boldsymbol{x}).\boldsymbol{y} = \boldsymbol{x}.(d\boldsymbol{y})$? Justify.

2. a. Find the quadratic equation $y = ax^2 + bx + c$ that passes through the points $(2, 0), (3, 5)$ and $(-1, -3)$.

   b. Use Gauss Jordan method to find complete solution set for the following homogeneous system of equations. Also give one particular solution.

$$2x_1 + 4x_2 - x_3 + 5x_4 + 2x_5 = 0$$
$$3x_1 + 3x_2 - x_3 + 3x_4 = 0$$
$$-5x_1 - 6x_2 + 2x_3 - 6x_4 - x_5 = 0$$

   c. Find the rank of the matrix $A = \left[\begin{array}{cccc|c} 2 & 1 & -2 & 0 & 1 \\ 2 & -4 & 3 & 1 & 0 \\ 3 & 15 & -13 & -2 & 7 \end{array}\right]$.

3. a. Use diagonalization method to calculate A^8 for

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}.$$

P.T.O.

b. Let $V = \mathbb{R}^2$ with the operations of addition and scalar multiplication defined as

$$[x,y] \oplus [w,z] = [x+w+1, y+z-2]$$

and
$$a \odot [x,y] = [ax+a-1, ay-2a+2]$$

respectively. Prove all the properties of a vector space involving only vector addition.

c. Prove or disprove that the set of all 2 x 2 symmetric matrices is a subspace of M_{22} under usual matrix operations.

4. a. Use simplified span method to find a simplified general form for all vectors in $span(S)$, where

$$S = \{x^3-1, x^2-x, x-1\}$$

is a subset of P_3, set of all polynomials of degree ≤ 3.

b. Show that

$$T = \left\{\begin{bmatrix}1 & -2\\ 0 & 1\end{bmatrix}, \begin{bmatrix}3 & 2\\ -6 & 1\end{bmatrix}, \begin{bmatrix}4 & -1\\ -5 & 2\end{bmatrix}, \begin{bmatrix}3 & -3\\ 0 & 0\end{bmatrix}\right\}$$

is a linearly independent subset of M_{22}. Does it form a basis of M_{22}?

c. Find the basis and dimension for the subspace W of $\mathbb{R}^3$ spanned by the set

$$S = \{[3,2,1], [1,2,0], [-1,2,-1]\}$$

of three vectors.

5. a. Check whether the following function is a linear transformation or not

$$T: P(\mathbb{R}) \rightarrow P(\mathbb{R}) \text{ defined by } T(f(x)) = x + \int_0^x f(t)dt$$

Also check that T is onto or not.

b. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation such that $T(1,0) = (1,4)$ and $T(1,1) = (2,5)$. Find a formula for $T(x,y)$ and also find $T(2,3)$.

c. Let T be a linear transformation from a vector space V to a vector space W. Prove that Kernel of T is a subspace of V and Range of T is a subspace of W.

6. a. State the Dimension Theorem for a linear transformation and verify it for linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(a,b) = (a+b,\ 0,\ 2a-b)$

b. Consider the linear transformation

$$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3 \text{ defined by}$$
$$T(v) = Av, \text{ where}$$
$$A = \begin{bmatrix}1 & -1 & 5\\ -2 & 3 & 13\\ 3 & -3 & 15\end{bmatrix}$$

Find the basis for kernel of T.

c. Define an Isomorphism between two vector spaces and check whether the map $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(a,b,c) = (3a-2c, b, 3a+4b)$ is an isomorphism or not?