[This question paper contains 4 printed pages.]

Your Roll No............

Sr. Number of the Question paper	:	**2009**
Unique Paper Code	**:**	**2223010045**
Name of the Paper	**:**	**Statistical Analysis in Physics**
Name of the Course	**:**	**B.Sc.(H) Physics**
Semester	**:**	**VI**

Duration : 2 Hours

Maximum Marks : 60

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt four questions in all.
3. Question number 1 is compulsory.
4. Non-programmable calculators are allowed.
5. Use of statistical tables are allowed.

1. Attempt any five of the following:

3*5=15 M

a) Differentiate between discrete and continuous probability distributions. Give one suitable example of each.
b) Let the joint probability density function of two random variables X and Y be given by $f(x,y) = kxy,\ 0 \leq x \leq 2\ 0 \leq y \leq 1$. Find: (i). The value of k (ii). $P(X < 1, Y < \frac{1}{2})$
c) A coin has probability θ of landing heads. Assume the prior distribution of θ is $\theta \sim Beta(3,1)$, The coin is tossed 5 times and 4 heads are observed. Determine the posterior distribution of θ and compute the posterior mean.
d) State Bayes' theorem. Briefly explain the relationship between prior probability and posterior probability with the help of a suitable example.
e) State the probability mass functions of the Binomial and Poisson distributions, and the probability density function of the Gaussian (Normal) distribution. Also write the mean and variance of each distribution.
f) Differentiate between Bayesian Regression and Ordinary Linear Regression. Discuss how uncertainty in model parameters is treated in both approaches.

2. (a) Two crystals are selected at random without replacement from a box containing 6 red, 7 green, and 9 white crystals. Let X denote the number of red crystals selected and Y denote the number of green crystals selected. Find:

(i) The joint probability distribution of X and Y

(ii) The marginal probability distributions of X and Y.

(iii) The conditional distribution of X given that $Y = 1$.

8M

(b) A continuous random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ x^2, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$$

Find:

(i) The probability density function of X.

(ii) $P(0.2 < X < 0.8)$.

(iii) The mean of the distribution.

(iv) The variance of the distribution.

7M

3. (a) L Let the joint probability density function of two continuous random variables X and Y be given by

$$f(x,y) = \begin{cases} 8xy, & 0 < x < 1, 0 < y < 1, x < y \\ 0, & otherwise. \end{cases}$$

Find:

(i) The marginal density functions $f_X(x) and\ f_Y(y)$.

(ii) $E(X)$ and $E(Y)$.

(iii) $Var(X)$ and $Var(Y)$.

(iv) $Cov(X, Y)$.

8M

(b) The marks obtained by students in a university examination have mean 65 and standard deviation 12. A random sample of 36 students is selected. Using the Central Limit Theorem, find:

(i) The mean and standard deviation of the sampling distribution of the sample mean.

(ii) The probability that the sample mean exceeds 68.

Assume that the sampling distribution of the sample mean is normal. Use standard normal tables wherever necessary.

3M+4M

4. (a) A screening test is used to detect HIV in a population. The following information is known:

(i) About 0.5% of the population is HIV positive.

(ii) If a person has HIV, the test correctly gives a positive result 99% of the time.

(iii) If a person does not have HIV, the test correctly gives a negative result 96% of the time.

A randomly selected person takes the test and receives a positive result. Find:

(i) The probability that the person is actually HIV positive.

(ii) The probability that the positive result is a false positive.

8M

(b) A factory has three machines, M_1, M_2 *and* M_3, producing electric components.

(i) Machine M_1 produces 40% of the total components, and 2% of its products are defective.

(ii) Machine M_2 produces 35% of the total components, and 3% of its products are defective.

(iii) Machine M_3 produces 25% of the total components, and 5% of its products are defective.

A component is selected at random and found to be defective. Find:

(i) The overall probability that a randomly selected component is defective.

(ii) The probability that the defective component was produced by machine M_1.

4M+3M

5. A coin has an unknown bias θ, representing the probability of landing heads. Assume that the prior distribution of θ is $\theta \sim Beta(3,2)$. The coin is tossed 6 times and 5 heads are observed.

(i) Determine the posterior distribution of θ.

(ii) Compute the posterior mean.

(iii) Compute the MAP estimate.

The coin is then tossed 4 more times, resulting in 1 head and 3 tails.

(iv) Update the posterior distribution again.

(v) Compute the new MAP estimate and briefly compare it with the previous estimate.

3+3+3+3+3M

6. A researcher models the relationship between study hours x and examination score y using the Bayesian linear regression model

$$y = \beta_0 + \beta_1 x + \epsilon$$

where

$$\epsilon \sim N(0, \sigma^2), \qquad \sigma^2 = 4$$

The prior distributions for the regression parameters are:

$$\beta_0 \sim N(0,10), \qquad \beta_1 \sim N(1,5)$$

A dataset consisting of the observations (1,4), (2,5), (3,8) is obtained.

(i) Write the likelihood function for the data.

(ii) Determine the posterior distribution of the parameter vector (β_0, β_1)

up to proportionality.

(iii) Explain how the posterior distribution combines prior belief and observed data.

(iv) Predict the expected score for a student studying $x = 4$ hours.

3+4+4+4M