$$f_n(x) = x^n, n \in N$$

is pointwise convergent but not uniformly convergent on [0, 1]. (2.5,5)

(b) State Weierstrass M-Test and show that the series:

$$\sum_{n=1}^{\infty} \frac{\sin(n^2 x^2)}{n^2}$$

is uniformly convergent on $(-\infty, \infty)$. (2.5,5)

(c) Find the radius of convergence of the power series: 7.5

$$\sum_{n=1}^{\infty} \frac{1}{n!} x^n .$$

6. (a) Show that the sequence $< f_n >$, where

$$f_n(x) = \frac{x}{1 + n^2 x^2}, \qquad n \in N$$

is uniformly convergent on [0,1]. 7.5

(b) Show that the series.

$$\sum_{n=1}^{\infty} \frac{x^n}{e^n}$$

is uniformly convergent on $(-\infty, \infty)$. 7.5

(c) Find the interval of convergence of the power series. 7.5

$$\sum_{n=1}^{\infty} \frac{x^n}{n(\log n)}$$

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This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **279 I**

Unique Paper Code : 2353510001

Name of the Paper : DSE: Elementary Mathematical Analysis

Name of the Course : **B.Sc. (Physical Science and Mathematical Science)**

Semester : VI

Time : 3 Hours ***Maximum Marks : 90***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **All** questions are compulsory.

(c) Attempt any **two** parts from each question.

(d) **All** questions carry equal marks.

(e) The use of calculator is **not** allowed.

1. (a) In each of the following, a function f(x) and a number x_0 are given. Use the sequential criterion to prove that $\lim_{x \to x_0} f(x)$ does not exist.

(i) $f(x) = \frac{|x|}{x}$; $\quad x_0 = 0$ 3.5

(ii) $f(x) = \begin{cases} 5, \text{ if } x < 3 \\ 6, \text{ if } x \geq 3 \end{cases}$; $\quad x_0 = 3$ 4

(b) Use the sequential criterion to find the limit of the function f(x) at x_0= 2 if it exist where

$$f(x)=\begin{cases} 1, \text{ if } x \text{ is rational} \\ 0, \text{ if } x \text{ is irrational} \end{cases}$$

and show that the function f(x)=sin(x) is uniformly continuous on $\mathbb{R}$. (3.5,4)

(c) Using the sequential criterion, show that if $\lim_{x\to x_0} f(x)=L$ and $\lim_{x\to x_0} g(x)=M$ then $\lim_{x\to x_0}(f(x)g(x))=LM$, where x_0 is a cluster point of domain of both functions f and g. 7.5

2. (a) Consider the function $T:\mathbb{R}\to\mathbb{R}$, defined by

$$T(x)=\begin{cases} \frac{1}{n} \text{ if } x=\frac{m}{n}, & \text{where } m\in\mathbb{Z}, n\in\mathbb{N} \text{ with } m \text{ and} \\ 1 \text{ if } x=0 & n \text{ have no common prime factors} \\ 0 \text{ if } x \text{ is irrational} & \end{cases}$$

Show that T(x) is continuous at every irrational number, but is discontinuous at every rational number. (7.5)

(b) Let f: [a,b] $\to$ [a,b] be a continuous function. Prove that there exists $c\in[a,b]$ such that f(c) = c 7.5

(c) Prove that $x2^x = 1$ for some $x\in(0,1)$. 7.5

3. (a) Define upper and lower (Darboux's) sums of f corresponding to the partition P of [a, b] for the function f(x) = x^2+3x

on [0, 2], find upper and lower sums for partition

$$P=\left\{0,\frac{1}{2},1,\frac{3}{2},2\right\}.$$ (3.5,4)

(b) If Q is a refinement of a partition P, then prove that

(i) $\underline{S}(f,Q)\ge\underline{S}(f,P)$ 3.5

(ii) $\overline{S}(f,Q)\le\overline{S}(f,P)$ 4

(c) If f is monotone on [a, b], then prove that f is integrable on [a, b]. 7.5

4. (a) Use sequential criterion for integrability to show that function f(x) = x^2-2x

is integrable on [2, 5] and also find $\int_2^5 f$. (5, 2.5)

(b) If f is integrable on [a, b], then prove that |f| is also integrable, further show that

$$\left|\int_a^b f\right|\le\int_a^b|f|\le M(b-a)$$

where, M is upper bound for |f| on [a,b]. Find a function f : [0,1] $\to\mathbb{R}$ such that f^2 is integrable on [0,1] but f is not. (2.5,1.5,3.5)

(c) State and prove the Fundamental Theorem of Calculus (first form). (1.5,6)

5. (a) Define pointwise convergent sequence and show that the sequence, $<f_n>$ where: