[This question paper contains 4 printed pages.]

**Your Roll No................**

**Sr. No. of Question Paper : 276** **L**

Unique Paper Code : 2353010013

Name of the Paper : Mathematical Finance

Name of the Course : **B.Sc. (Hons.) Mathematics – DSE**

Semester : VI

Duration : 3 Hours Maximum Marks : 90

**<u>Instructions for Candidates</u>**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory and carry equal marks.
4. Use of scientific calculator, basic calculator and normal distribution tables all are allowed.

**1** Attempt any two parts:

(a) Define the convexity of a bond and find the relation between the Convexity, Duration and Bond price. How does convexity measure sensitivity of the portfolio?

(b) Derive the following relation:

$$R_F = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

where $R_F$ is the forward interest rate for the period between $T_1$ and $T_2$, $R_1$ and $R_2$ are the zero rates for maturities $T_1$ and $T_2$ respectively. Calculate the year-4 forward rate from the data:

$$T_1 = 3, T_2 = 4, R_1 = 0.046, \text{ and } R_2 = 0.05$$

(c ) Explain Duration of a zero- coupon bond. Consider a 5-year bond with a yield of 8% (continuously compounded) that pays a 7% annual coupon at the end of each year.

(i) What is the bond's price?

(ii) Use duration to estimate the effect on the bond's price of a 0.2% increase in its yield. (Use $e^x$ values: $e^x = 0.9231, 0.8521, 0.7866$ for $x = -0.08, -0.16, -0.24$ respectively.)

2 Attempt any two parts:

(a)Define Derivative and present a comparative study for Future and Forward Derivative. Investor shorts 50,000 pounds forward contract for U.S dollars at an exchange rate of 1.4000 USD per pound. Find gain or loss if exchange rate at the end of the contract is (i)1.3800 (ii) 1.4200.

(b) (i)Present a comparative study for different types of traders in the financial markets?

(ii) A portfolio contains 10,000 shares worth $50 each. How can put options be used to provide insurance against a decline in the value of the holding over the next 4 month.

(c) A one-year forward contract on a non-dividend paying stock is· entered into when the stock price is 50, and the risk-free rate of interest is 8% per annum with continuous compounding. What is the forward price? Justify using no arbitrage arguments.

3 Attempt any two parts:

(a) List the four types of option positions. Illustrate each with their respective payoff functions and payoff diagrams from positions in European options.

(b) Define the upper bound and lower bound for European options on a non-dividend- paying stock. Also, what is a lower bound for the price of a 2-month European put option on a non-dividend-paying stock when the stock price is $58, the strike price is $65, and the risk-free interest rate is 5% per annum? (You can use the exponential value $e^{-0.0083} = 0.9916$).

(c)Derive the relationship between the prices of European put and call options that have the same strike price and time to maturity, assuming the underlying stock pays no dividends. Using this relationship, determine the price of a 3-month European call option when the price of a non-dividend-paying stock is \$31, the price of a 3-month European put option on the stock with a strike price of \$30 is \$1.259 and the risk-free rate is 10% per annum. (You can use the exponential value $e^{-0.025} = 0.9753$).

4 Attempt any two parts:

(a)Explain how a bear spread can be created using European put options on a stock. Also, present the payoff table and draw the corresponding profit diagram for this strategy. Call options on a stock are available with strike prices of \$15, $\$17\frac{1}{2}$, and \$20, and expiration dates in 3 months. Their prices are \$4, \$2, and $\$\frac{1}{2}$, respectively. Explain how the options can be used to create a butterfly spread. Create a table showing how profit varies with stock price for the butterfly spread.

(b)A stock price is currently \$80. It is known that at the end of 4 months it will be either \$75 or \$85. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a 4-month European put option with a strike price of \$80? Use no- arbitrage arguments. (You can use the exponential value $e^{0.0167} = 1.0168$).

(c)A stock price is currently \$50. It is known that at the end of 6 months it will be either \$60 or \$42. The risk-free rate of interest with continuous compounding is 12% per annum. Calculate the value of a 6-month European call option on the stock with an exercise price of \$48. Verify that no-arbitrage arguments and risk-neutral valuation arguments give the same answers. (You can use the exponential value $e^{0.06} = 1.0618$).

5 Attempt any two parts:

(a)Define the delta of an option. Calculate the delta for European call and put option when the stock price is Rs 1225, the strike price is Rs 1250, the risk-free interest rate is 5% per annum, the stock price volatility is 20% per annum and the time to maturity is 20 weeks.

(b)A possible sequence of the stock prices during 8 consecutive trading days is

$$100, 102, 99, 100, 103, 101, 105, 107$$

Estimate the stock price volatility. What is the standard error of your estimate?

(c)Consider a variable $S$ that follows the process $dS = \mu\, dt + \sigma\, dz$.For the first three years, $\mu = 2$ and $\sigma = 3$; for the next three years, $\mu = 3$ and $\sigma = 4$. If the initial value of the variable is 10, what is the probability distribution of the value of the variable at the end of year six?

6 Attempt any two parts:

(a)Suppose that a financial institution has following three positions in options on a stock: (i) A long position in 1000 call options with strike price Rs 1250 and an expiration date in 4-months. The delta of each option is 0.542. (ii) A short position in 2000 call options with strike price Rs 1300 and an expiration date in 3-months. The delta of each option is 0.486. (iii) A short position in 500 put options with strike price Rs 1310 and an expiration date 2-months. The delta of each option is $-0.500$. Calculate the delta of the portfolio of options. How can you make portfolio delta neutral?

(b)Define the gamma of an option. For a non-dividend-paying stock, use the put–call parity relationship to derive the relationship between the gamma of European call and the gamma of European put.

(c)What is the price of a European call option on a non-dividend-paying stock when the stock price is Rs 42, the strike price is Rs 40, the risk-free interest rate is 10% per annum, the stock price volatility is 20% per annum and the time to maturity is 6 months?