[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1798 **L**

Unique Paper Code : 2372011201

Name of the Paper : DSC: Theory of Probability Distributions (NEP)

Name of the Course : **B.Sc. (Hons) Statistics (DSC)**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all, selecting **three** questions from each section. **All** questions/parts are of equal marks. All notations used have their usual meanings.

Section – A

1. (a) Two independent random variables X and Y assume identical values – 1, 1, 3 with equal probabilities, then find $M_{X+Y}(t)$ and hence compute the distribution of X + Y.

   (b) Let X and Y have joint probability density function

$$f(x,y)=\begin{cases}\frac{1}{2}, & 0<y<x<2\\ 0, & \text{otherwise.}\end{cases}$$

   Compute E(Y|X), and Var(E(Y|X)).

2. (a) Find characteristic function of the random variable whose moments about origin are given by $\mu_r' = (r+1)!\,9^r,\ r=1,2,\ldots$.

   (b) Define cumulant generating function and show that all cumulants of Poisson distribution are equal.

3. (a) Let X and Y have joint probability density function

$$f(x,y)=\begin{cases}e^{-(x+y)}, & 0<x,y<\infty\\ 0, & \text{otherwise.}\end{cases}$$

   Find the distribution of X – Y.

   (b) If X and Y are independent normal variates with means – 1, 2 and variances 4 and 9 respectively, then determine k such that $P(3X+2Y\le k)=P(X-4Y\ge k)$.

4. (a) For the probability mass function $P(X=x)=2\left(\frac{1}{3}\right)^{x+1}, x=0,1,2,\ldots,$ compute $F_X(x)=P(X\le x)$ and hence $E(F_X(x))$.

(b) Show that $\mu_r' = \sum_{j=1}^{r} \binom{r-1}{j-1} \mu_{r-j}' \kappa_j$.

Section – B

5. (a) If X and Y are independent Poisson variates with means λ_1 and λ_2 respectively, then find conditional distribution of $X|(X+Y=s)$.

(b) Show that for Poisson distribution with mean (λ), $\mu_{r+1}=\lambda\left(r\mu_{r-1}+\frac{d\mu_r}{d\lambda}\right), r=1,2,..$ and hence find first four central moments.

6. (a) In tri-variate distribution, derive an equation of plane of regression of X_1 on X_2 and X_3.

(b) In tri-variate distribution show that $\sigma_{1.23}^2=\sigma_1^2\frac{\omega}{\omega_{11}}$.

7. (a) If a fair coin is tossed an even number (2k) times. Show that the probability of obtaining more tails than heads is $\frac{1}{2}\left[1-\binom{2k}{k}\left(\frac{1}{2}\right)^{2k}\right]$.

(b) Show that for $N(0,\sigma^2)$ distribution $\mu_{2r+1}=0$ and $\mu_{2r}=1\times3\times5\times\ldots\times(2r-1)\sigma^{2r}$.

8. (a) Find mode of Binomial distribution with parameters (n, p).

(b) What is mode of Binomial distribution in the following cases

(i) Binomial distribution with parameters n = 12 and p = 1/3 and

(ii) Binomial distribution with parameters n = 15 and p = 1/4 .