

[This question paper contains 4 printed pages.]

Your Roll No.....

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Sr. No. of Question Paper : 1793
Unique Paper Code : 2352011201
Name of the Paper : Linear Algebra
Name of the Course : B.Sc. (H) Mathematics
Semester : II / DSC
Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting any **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculators is not allowed.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that

$$\|x + y\| \geq |\|x\| - \|y\||$$

Also verify it for the vectors $x = [2, -1, 3, 2]$ and $y = [4, 3, 2, 1]$ in $\mathbb{R}^4$.

(5+2.5)

- (b) Solve the following system of linear equations using Gaussian Elimination method :

$$3x_1 + x_2 + 7x_3 + 2x_4 = 13$$

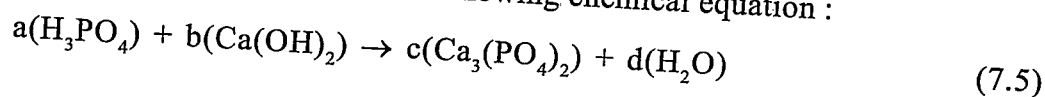
$$2x_1 - 4x_2 + 14x_3 - x_4 = -10$$

$$5x_1 + 11x_2 - 7x_3 + 8x_4 = 59$$

$$2x_1 + 5x_2 - 4x_3 - 3x_4 = 39$$

Also give the complete solution set if the system is consistent. (6+1.5)

- (c) Using the Gauss-Jordan method, find the minimal integer values for the variables that will balance the following chemical equation :



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2. (a) Find the reduced row echelon form B of the following matrix A, keeping track of the row operations used :

$$A = \begin{bmatrix} 4 & 0 & -20 \\ -2 & 0 & 11 \\ 3 & 1 & -15 \end{bmatrix} \quad (7.5)$$

- (b) Determine whether the vector $x = [5, 17, -20]$ is in the row space of the matrix

$$A = \begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ -2 & 4 & -3 \end{bmatrix}$$

If so, then express $[5, 17, -20]$ as a linear combination of the rows of A. (5+2.5)

- (c) (i) Define Eigenvalue and Eigenspace for a square matrix A.
(ii) Find all the eigenvalues and eigenspaces for A.

$$A = \begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix} \quad (2+5.5)$$

3. (a) Define a subspace of a vector space over $\mathbb{R}$. Which of the following are subspaces of $M_{2 \times 2}(\mathbb{R})$? Give reasons to support your answer.

(i) $W_1 = \{A \in M_{2 \times 2} : \text{Trace}(A) = 0\}$

(ii) $W_2 = \{A \in M_{2 \times 2} : A^T = 4I\}$

(iii) $W_3 = \{A \in M_{2 \times 2} : |A| = 0\}$

(1.5+2+2+2)

- (b) Define sum of two subspaces W_1 and W_2 of a vector space $V(\mathbb{R})$. Show that $W_1 + W_2$ is the smallest subspace of V containing both W_1 and W_2 . (2+5.5)

- (c) Define Linear Span of a set S in a vector space V . Is the linear span of S a subspace of V ? Determine if the vector $(2, -1, 1)$ is in the linear span of $S = \{(1, 0, 2), (-1, 1, 1)\}$ (2+2+3.5)
4. (a) Define a linearly independent subset of a vector space. For $k = 0, 1, 2, \dots, n$, $p_k(x) = x^k + x^{k+1} + \dots + x^n$. Show that $\{p_0(x), p_1(x), \dots, p_n(x)\}$ is linearly independent subset of the space of all polynomials. (2+5.5)
- (b) Let $W = \{(a_1, a_2, a_3, a_4, a_5) : a_2 = a_4, a_1 + a_3 + a_5 = 0\}$ be a subset of $\mathbb{R}^5$. Is W a subspace of $\mathbb{R}^5$? Find a basis for W . What is $\dim_{\mathbb{R}} W$? (2+4+1.5)
- (c) Let $S = \{(1, 3, -4, 2), (2, 2, -4, 0), (1, -3, 2, -4), (-1, 0, 1, 0)\}$ be a subset of $\mathbb{R}^4$. Show that S is linearly dependent. Write a vector in S as a linear combination of other vectors. (5.5+2)
5. (a) Define a linear transformation. Is there any linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ such that $T(1, 0, 3) = (1, 1)$ and $T(-2, 0, -6) = (2, 1)$? (2+5.5)
- (b) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by $T(a, b) = (a - b, a, 2a + b)$
- Let β be the standard ordered basis for $\mathbb{R}^2$ and $\gamma = \{(1, 1, 0), (0, 1, 1), (2, 2, 3)\}$. Compute $[T]_{\gamma}^{\beta}$. (7.5)
- (c) (i) Let V and W be vector spaces and let $T: V \rightarrow W$ be a linear transformation. Then show that T is one-to-one if and only if $N(T) = \{0\}$.
- (ii) Let $T: P_2(\mathbb{R}) \rightarrow P_3(\mathbb{R})$ be a linear transformation defined by
- $$T(f(x)) = 2f'(x) + \int_0^x 3f(t)dt.$$
- Show that T is one-one. (3+4.5)
6. (a) Let V and W be vector spaces and let $T: V \rightarrow W$ be a linear. Define inverse of T . If T is invertible linear transformation then show that $T^{-1}: W \rightarrow V$ is linear. (2+5.5)

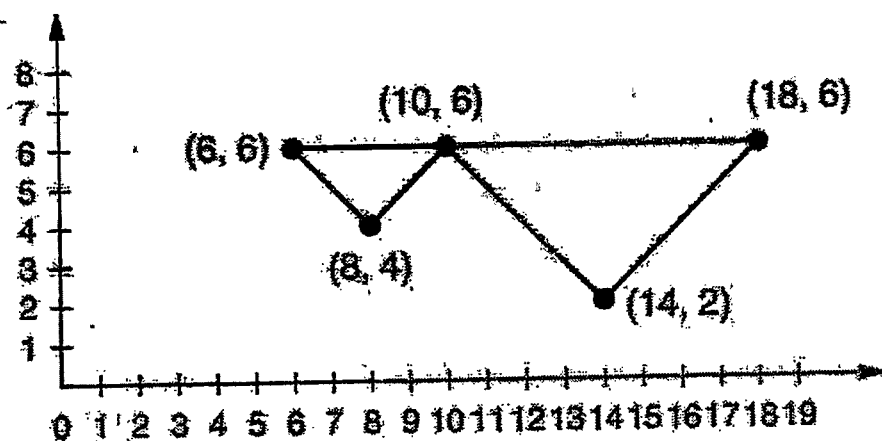
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(b) Define isomorphic vector spaces. Which of the following spaces are isomorphic? Justify your answer.

(i) $M_{2 \times 2}(\mathbb{R})$ and $P_3(\mathbb{R})$

(ii) $V = \{A \in M_{2 \times 2}(\mathbb{R}) : \text{tr}(A) = 0\}$ and $\mathbb{R}^4$ (2.5+2.5+2+2.5)

(c) For the graphic in given figure, use ordinary coordinates to find the new vertices after performing each indicated operations :



(i) Rotation about the origin through $\theta = 90^\circ$

(ii) Reflection about the line $y = \frac{1}{2}x$.

(iii) Translation along the vector $[-3, 5]$ (2.5+2.5+2.5)