[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1884 **L**

Unique Paper Code : 2353012006

Name of the Paper : Mechanics (UGCF)

Name of the Course : **B.Sc. (H) Mathematics – DSE**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. Part of the questions to be attempted together.
4. Use of Calculator not allowed.
5. All questions carry **15** marks.

Q1.

a). Three forces P, Q, R acting on a particle are in equilibrium and the angle between P and Q is double of the angle between P and R. Show that

$$P = \frac{(Q^2 - R^2)}{Q}$$

b). State the Principle of Virtual Work. A framework ABCD consists of four equal, light rods smoothly jointed together to form a square; it is suspended from a peg at A, and a weight W is attached to C, the framework being kept in shape by a light rod connecting B and D. Determine the thrust in this rod.

c). (i). Any force system in the fundamental plane may be reduced either to a single force or to a couple.

(ii). Find the co-ordinates of the center of the gravity of a quadrant of the area of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ bounded by the axes of co-ordinates.

Q2.

a). State and prove Pappus Theorem for the mass center of a uniform distribution along a plane curve. Hence, find the mass center of a wire bent into the form of an isosceles right-angled triangle.

b). A light ladder is supported on a rough floor and leans against a smooth wall. How far up the ladder can a man climb without slipping taking place?

c). (i) Show that a field of force with components (X, Y) is conservative if and only if, $\frac{\partial X}{\partial y} = \frac{\partial Y}{\partial x}$.

(ii). Forces P, Q, R act along the lines x = 0, y = 0, x.cosθ+y.sinθ = p. Find the magnitude of the resultant and the equation of its line of action?

Q3.

a). Prove that for a particle moving in a plane and occupying the position $P(r,\theta))$ at a time t, the radial and transverse components of velocity are $\left(\frac{dr}{dt}, r\frac{d\theta}{dt}\right)$ and radial and transverse components of acceleration are $(\ddot{r} - r\dot{\theta}^2, \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}))$.

b). (i) Prove that the motion of the center of mass of a system of particles is equivalent to the motion of a single particle of mass equal to the total mass of the system, subject to the net external force acting on the system.
(ii) State D'Alembert's Principle. Explain why D'Alembert's principle gives the impression that it reduces dynamics to statics.

c). A damped harmonic oscillator is subjected to an external periodic force, and its motion is governed by the equation:

$$m\frac{d^2x}{dt^2} + 2m\mu\frac{dx}{dt} + mp^2x = mk\cos(ct)$$

where m is the mass, μ is the damping coefficient, p is the natural frequency, k is the amplitude of the external force per unit mass, and c is the angular frequency of the driving force. Derive the particular solution of the above equation for steady-state motion. Hence, show that when the driving frequency equals the natural frequency i.e., $c = p$, the solution reduces to

$$x(t) = \frac{k}{2\mu p} cos\left(pt - \frac{\pi}{2}\right).$$

Q4.

a). Derive the general equation of the orbit under a central inverse-square law force, and show that the orbit is a conic section given by

$$u = \frac{1}{l}(1 + e\, cos\theta)$$

where l is the semi-latus rectum, and e is the eccentricity. Also, state what types of orbits are represented by the conditions $e < 1$, $e = 1$, and $e > 1$.

b). Derive the equation of path of a particle which is projected in a medium having no resistance. Show that the path is a parabola. Find its vertex and latus rectum. Also find its maximum horizontal range.

c). Define an inertial frame of reference. Show that the laws of motion are invariant under transformation from one inertial frame to another moving with uniform translational velocity. Given one inertial frame of reference, how many inertial frames of reference can exist? Justify your answer.

Q5.

a). A particle of mass m oscillates in a line with natural period $\frac{2\pi}{n}$. If an applied force F Cos (p t), now acts in the line so that the particle is instantaneously at rest at zero time at a distance d from the centre. Prove that the displacement of the particle at a subsequent time t is

$$d\, Cos\, n\, t + \frac{F}{(n^2 - p^2)\, m}(Cos\, p\, t - Cos\, n\, t).$$

b). Show that the equation of the trajectory of the projectile $y = x \tan\alpha - \frac{g\, x^2}{2\, u^2\, Cos^2\, \alpha}$, and hence show that the horizontal range of the projectile is $R = \frac{u^2\, Sin\, 2\alpha}{g}$.

c). A particle is projected with initial velocity V from a point O at angle of elevation α. Show that the range on a plane through O inclined at β to the horizontal ($\beta < \alpha < \frac{\pi}{2}$) is $\frac{V^2}{g\, Cos^2 \beta}[\, Sin\,(2\alpha - \beta) - Sin\,\beta\,]$.

Q6.

a). Define specific gravity, and determine the specific gravity of mixture of a given weight of different substances whose specific gravities are given. Hence evaluate the specific gravity of the oil where the weight of an empty vessel is 3 oz; when filled with water it is 9 oz and when filled with oil it is 8.4 oz.

b). State compressibility. A mixture of oxygen and nitrogen is found to be consists 21 parts of oxygen to 79 of nitrogen by volume, or by weight 23 parts to 77. Compare the densities of the gases.

c). A circular tube with center O is filled with fluids of densities ρ_1, ρ_2 & ρ_3 in descending order of magnitude [i.e. $\rho_1 > \rho_2 > \rho_3$] is placed in a vertical plane. If $2\alpha, 2\beta$ & 2γ be the angle subtended at the center by fluids and P be the point on the circumference midway between the ends of the lightest fluid, then the angle θ which OP makes with the vertical is given by $\frac{\rho_2 - \rho_3}{\rho_1 - \rho_3} = \frac{Sin\,\alpha\, Sin\,(\beta-\theta)}{Sin\,(\theta+\alpha)\, Sin\,\beta}$.