[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1882 **L**

Unique Paper Code : 2353012004

Name of the Paper : Biomathematics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. Each part of questions is of **7.5** marks.

1.

(a) Write the logistic growth model and find out the equilibrium points. Also, classify them as stable or unstable.

(b) A drug is available in doses of M mg, which raise the blood plasma concentration of an average adult by 10 mgl^{-1}. Once in the blood plasma, the drug concentration decays according to the equation

$$\frac{dC(t)}{dt} = \frac{-C(t)}{\tau} \text{ with } \tau = 4h.$$

Health regulations require that concentration never exceeds 15 mgl^{-1}. What is the shortest safe time interval at which dose can be given regularly? What is the minimum concentration if doses are given at this time interval?

(c) The rate of increase of bacteria in a culture is proportional to the number present. The population multiplies by the factor n in the time interval T. Find the number of bacteria at time t when the initial population is P_0.

2.

(a) State the law of mass action. Use the law to derive a system of equations modelling the chemical reaction

$$A + Y \xrightarrow{k_1} X,$$
$$X + Y \xrightarrow{k_2} P,$$
$$A + X \xrightarrow{k_3} 2X + Z,$$
$$2X \xrightarrow{k_4} Q$$
$$Z \xrightarrow{k_5} fY.$$

(b) Describe the Verhulst-model for the discrete population growth. Show that the equilibrium states for this model are $P = 0$ and $P = K$.

(c) Determine the rest states in Volterra-Lotka model of predator-prey interaction

$$\frac{dX}{dt} = -eX + fXY$$
$$\frac{dY}{dt} = aY - bY^2 - cXY.$$

If predator and prey are present in the steady state, show that the coefficients *a, b, c, e* and *f* must satisfy the constraints

$$\frac{a}{b} > \frac{e}{f}, c \neq 0.$$

3.

(a) Discuss the direction of travel near the null-clines of the following modified SIS model:

$$\frac{dS}{dT} = -0.002\,SI + 0.1\,S,$$

$$\frac{dI}{dT} = 0.002\,SI - 0.4\,I.$$

Also sketch the phase plane diagram.

(b) Explain the SIS (Susceptible-Infectives-Susceptible) epidemic model. State its assumptions, write the differential equations, and explain the parameters used.

(c) Describe the SIR (Susceptible-Infectives-Recovered) epidemic model with schematic representation and show that the epidemic outbreak will not take place if $S(0) < \frac{\beta}{\alpha}$, where α is the infection rate and β is the per-capita recovery rate.

4.

(a) Consider the predator–prey model:

$$\frac{dV}{dt} = \alpha\left(1 - \frac{V}{K}\right)V - \gamma OV, \quad \beta = \frac{\alpha}{K},$$

$$\frac{dO}{dt} = -\delta O + \epsilon OV.$$

Here $V(t)$ stands for number of Voles (prey) and $O(t)$ stands for number of Owls (predator) at time t. K is the carrying capacity of prey growth. The other parameters have usual meanings. Show that the equilibrium point $\left(\frac{\delta}{\epsilon}, \frac{\alpha}{\gamma}, -\frac{\beta\delta}{\gamma\epsilon}\right)$ is asymptotically stable when $\frac{\delta}{\epsilon} < \frac{\alpha}{\beta}$.

(b) Determine the nature and stability of the equilibrium point (0, 0) for the system:

$$\dot{x} = 5x + 2y,$$
$$\dot{y} = 2x + 2y,$$

by determining the eigenvalues of the Jacobian matrix associated with the equilibrium point.

(c) Use Poincaré–Bendixson theorem to show that the system

$$\frac{dV}{dt} = f(V,O) = \frac{2}{3}V - \frac{V^2}{6} - \frac{VO}{1+V},$$

$$'\frac{dO}{dt} = g(V,O) = \delta O\left(1 - \frac{O}{V}\right),$$

has a periodic solution for $0 < \delta < \frac{1}{12}$.

5. (a) What is bifurcation? Show that

$$\frac{dx}{dt} = x(y-1),$$
$$\frac{dy}{dt} = \mu - y(x+1),$$

has an equilibrium point, which is a stable node, for $\mu < 1$, that becomes a saddle-point as μ passes through the bifurcation point $\mu = 1$. Also show that there is an additional equilibrium point when $\mu > 1$, which is a stable node.

(b) What is Hopf bifurcation? Show that Hopf bifurcation holds for the following system

$$\dot{x} = -y + x(\mu - x^2 - y^2),$$
$$\dot{y} = x + y(\mu - x^2 - y^2).$$

(c) Discuss the stability of limit cycle when $P_0, P_1, P_2, \ldots$ are the points on Poincare line and P_n is at a distance of s_n form P_0 and such that $s_{n+1} = f(s_n)$, where P_0 is the fixed point.

6. (a) Show that

$$\dot{x} = \mu x(2y-1),$$
$$\dot{y} = \mu - y(2x+1),$$

has an equilibrium point, which is a saddle node for $\frac{1}{2} < \mu < 1$ and a saddle focus for $\mu > 1$. Prove that $\mu = \frac{1}{2}$ is a bifurcation point.

(b) Write the transition matrix for Jukes-Cantor model of base substitution stating the significance of each entry. In what proportion of the sites will each base appear after one time step? What proportion of sites will have a base **A** in the ancestral sequence and a base **T** in the descendent sequence one step later?

(c) Explain Kimura 2-parameter and 3-parameter models along with their corresponding distance formulas. Write the expression of the log-det distance between S_0 and S_1.