

2977

12

What is the minimum number of time periods required to schedule all six club meetings so that no student has a conflict?

[This question paper contains 12 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2977

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Unique Paper Code : 2353572005

Name of the Paper : Introduction to Graph Theory

Name of the Course : **DSE**

Semester : IV

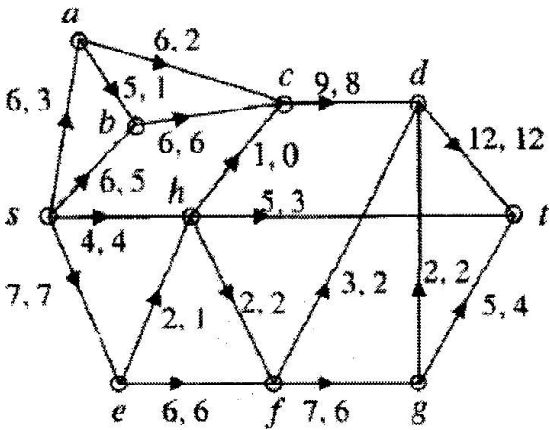
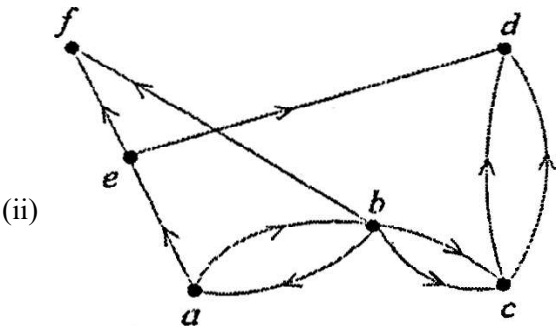
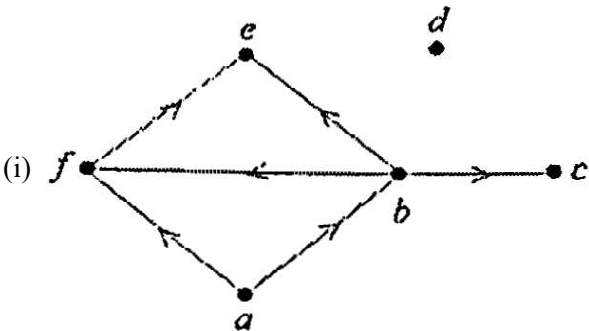
Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **Both** parts of a question to be attempted together.
4. **All** questions carry equal marks (**7.5** marks each).

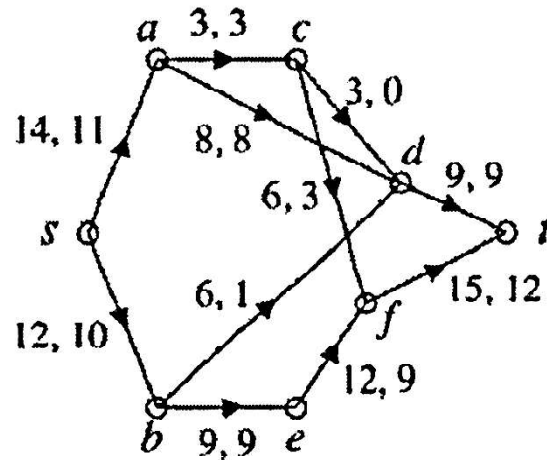
1. (a) Define Degree of a Vertex. Find the Indegree and Outdegree of each Vertex in the following directed graphs.



- (c) A university needs to schedule introductory meetings for six different student clubs. Some students belong to multiple clubs, so those clubs cannot meet at the same time.

Clubs	Members
Robotics Club	Alice, Bob, Charlie
Art Collective	Bob, Daisy, Ethan
Debate Team	Charlie, Daisy, Fiona
Coding Club	Alice, Ethan, George
Music Ensemble	Fiona, George, Hannah
Chess Club	Alice, Hannah

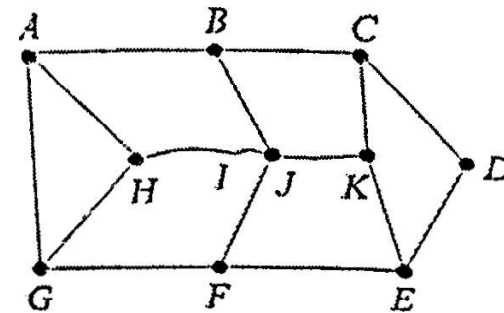
6. (a) Verify the law of conservation of flow at a, c, and d. Find the value of indicated flow and also determine the capacity of the (s, t)-cut defined by $S = \{s, a, b, d\}$ and $T = \{c, e, f, t\}$.



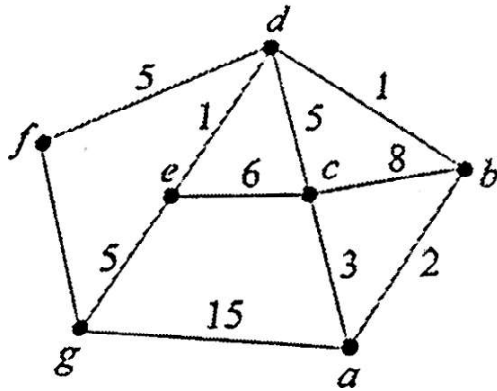
- (b) State the Max flow-Min Cut theorem and verify it for the given below graph.

Also draw a graph with 5 vertices and as many edges as possible. How many edges does your graph contain? What is the name of this graph and how is it denoted?

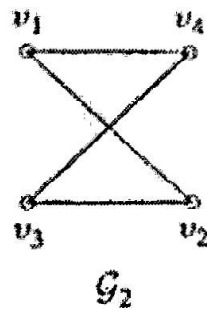
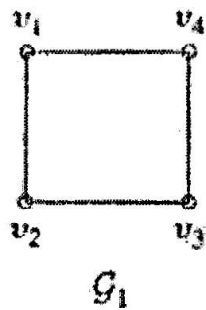
- (b) What is Hamiltonian graph? Is the following graph Hamiltonian, explain. Find Hamiltonian path in the following graph. Also define Eulerian Graph with examples. Give an example of a graph which is Eulerian as well as Hamiltonian.



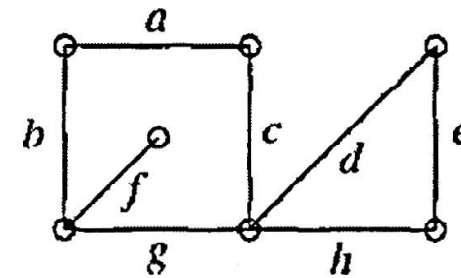
- (c) Use Dijkstras Algorithm, find the shortest path between the vertices a and g in the following Graph.



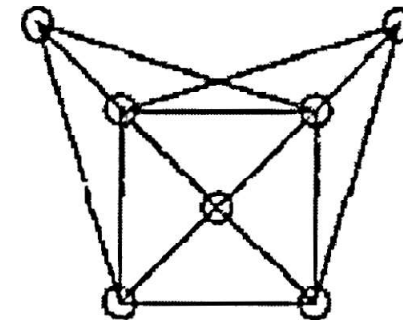
2. (a) Find the Adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find a Permutation matrix P such that $A_2 = PA_1P^T$. Prove that G_1 and G_2 are Isomorphic.



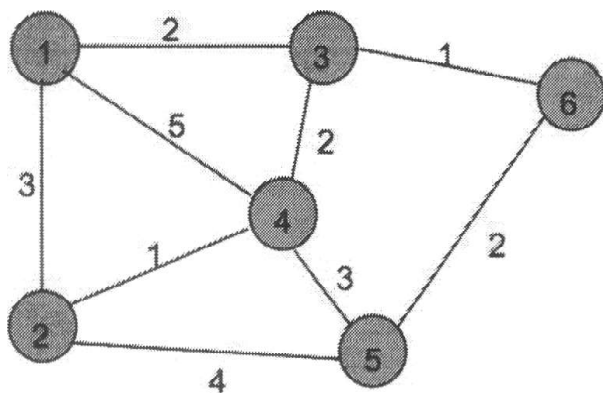
- (b) For the given graph, find the number of regions. List all the edges which form the boundary of each region and verify the Euler's formula.



- (c) Define a chromatic number of the graph. Find a coloring which requires the least number of colors for the given graph, hence find its chromatic number.



- (b) Apply Prim's algorithm to the above given graph in (a) to find the minimum spanning tree and compare results.
- (c) Apply Bellman's algorithm to the given digraph, to find the length of the shortest path from node 2 to every other vertex.

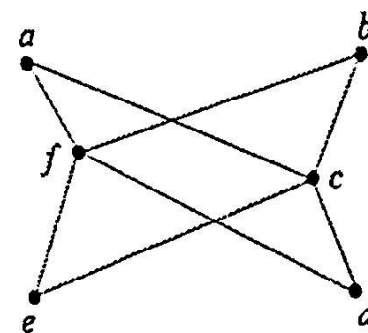


5. (a) Define the Planar graph. Show that $K_{3,3}$ is not planar.

- (b) Solve the following Traveling Salesman Problem. The distances from one city to another city are given in the following table:

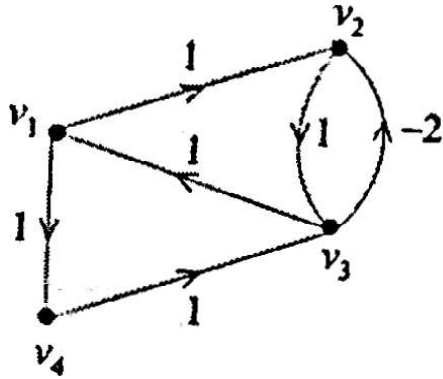
	A	B	C	D	E
A	-	2	12	4	6
B	6	-	10	4	6
C	12	10	-	12	8
D	4	4	12	-	12
E	6	6	8	12	-

- (c) Define a Bipartite graph, hence or otherwise determine whether the following graph is bipartite. Give reasons in support of your answer.

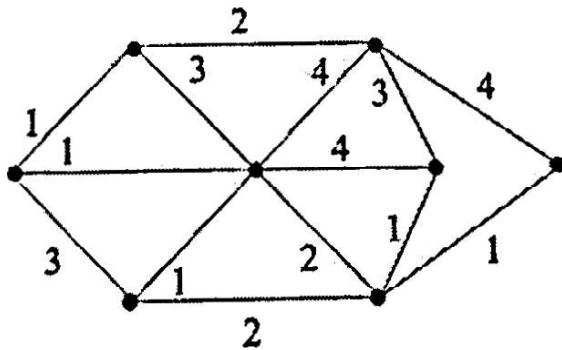


Define a Complete bipartite Graph. Draw $K_{2,3}$. Is it a Bipartite or a Complete Bipartite Graph?

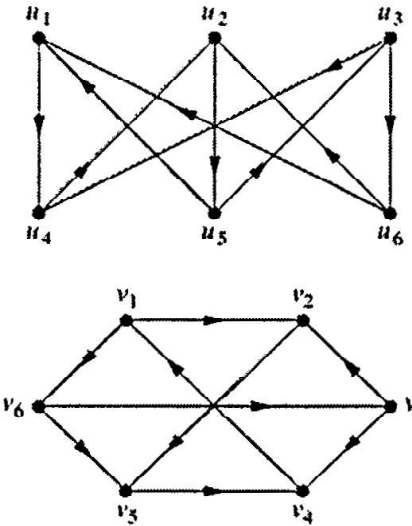
3. (a) Apply Beliman-Ford algorithm in the digraph given below to find the length of shortest path from vertex s to every other vertex.



- (b) Solve the Chinese Postman problem for the given weighted graph.



- (c) Determine whether the given pair of digraphs are isomorphic.



4. (a) Find minimum spanning tree of the following weighted graph using Kruskals algorithm.

