[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2976 **L**

Unique Paper Code : 2353572004

Name of the Paper : Elements of Discrete Mathematics

Name of the Course : **B.Sc. (Physical Sciences, Mathematical Science)-DSE**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. Part of the questions to be attempted together.
4. **All** questions carry equal marks.

1. (a) Define maximal and greatest element in a poset $(A, \leq)$. Prove that every greatest element is maximal. Give an example of a poset which has exactly one maximal element but does not have a greatest element. (7.5)

(b) Prove by mathematical induction, that $n^3 - n$ is divisible by 3 for every positive integer n. (7.5)

(c) Define order isomorphism between two posets. Let $A = (Z^+, \leq)$ and let $B = (2Z^+, \leq)$ be the set of positive even integers together with usual $\leq$ relation. Prove that A and B are ordered isomorphic (7.5)

2. (a) Define an algebraic lattice. Let $(L, \leq)$ be a lattice ordered set, define $x \wedge y = inf\{x, y\}$ and $x \vee y = \sup\{x, y\}$. Prove that $(L, \wedge, \vee)$ is an algebraic lattice. (7.5)

(b) Let $A = \{2, 3, 5, 6, 10, 15, 30\}$. Define the relation $\leq$ on A as $m \leq n$ if m *divides* n. Draw a Hasse diagram of the ordered set $(A, \leq)$. Is $(A, \leq)$ lattice ordered set? Justify. (7.5)

(c) Define the Tautology and Contradiction statements. If each statement p and q are true, then find the truth value of $(\sim p \wedge (p \vee q)) \Rightarrow q$. (7.5)

3. (a) Prove that a lattice L is distributive if and only if $\forall\, a, b, c \in L$ we have
$(a \vee b = c \vee b \text{ and } a \wedge b = c \wedge b) \Rightarrow a = c$. (7.5)

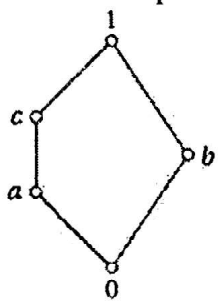
(b) Show that the lattice ({1,2,3,6,9,18}, gcd, lcm), consisting of the positive divisors of 18, is not a Boolean algebra. (7.5)

(c) Draw the Hasse diagram of power set P(S) of the set $S = \{a, b, c\}$ under the "contained in" relation. How many elements are there in P(S)? Is it a chain? Identify the least and greatest elements. (7.5)

4. (a) Prove that product of two posets is a poset. (7.5)

(b) Prove that the image of a distributive lattice under a homomorphism is also a distributive lattice. (7.5)

(c) Define complemented lattice. Consider the Hasse diagram given below



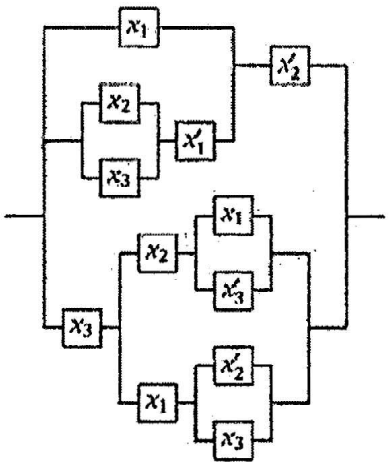
Is it a complemented lattice? If yes, write complement of all the elements. (7.5)

5. (a) Simplify the following Boolean polynomials

(i) $(x \wedge y) \vee (x \wedge y') \vee (x' \wedge y)$

(ii) $(x \wedge y') \vee (x \wedge (y \wedge z)') \vee z$ (7.5)

(b) Find the symbolic gate representation of the contact diagram given below:



(7.5)

(c) Using Quine Mc-Cluskey method, find the minimal form of the Boolean polynomial d whose disjunctive normal form is given as follows:

$d = wxyz' + wxy'z' + wx'yz + wx'yz' + w'x'yz + w'x'yz' + w'x'y'z.$ (7.5)

6. (a) Use Karnaugh diagram to simplify the polynomial

$p = (x_1 + x_2)(x_1 + x_3) + x_1x_2x_3$ (7.5)

(b) A voting machine for three voters has three YES-NO switches. Current is in the circuit precisely when YES has a majority. Draw the contact diagram and the symbolic representation by gates and simplify it. (7.5)

(c) Find the Disjunctive Normal and Conjunctive Normal form of the Boolean polynomial $(a + b + c)(ab + a'c)'$. (7.5)