[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2462 **L**

Unique Paper Code : 2372572403

Name of the Paper : DSC: Basics of Statistical Inference

Name of the Course : **B.Sc. (P)/ B.A. (P) with Statistics under UGCF**

Semester : IV (NEP)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **Six** questions. **All** questions carry equal marks.
3. Use of non-programmable scientific calculator is allowed.

1. (a) Define an unbiased estimator of a parameter. Let X follows Poisson(θ) distribution. Show that the only unbiased estimator of $e^{\{-(k+1)\theta\}}, k > 0$ is $T(x) = (-K)^x$, so that

$$T(x) > 0 \text{ if } x \text{ is even}$$
$$T(x) < 0 \text{ if } x \text{ is odd.}$$

(b) Prove that for Cauchy distribution, not sample mean but sample median is a consistent estimator of the population mean. **(8, 7)**

2. (a) Define sufficiency of an estimator. Let $X_1, X_2, ..., X_n$ be a random sample from uniform population on $(0, \theta)$. Find a sufficient statistic for θ.

(b) Prove that minimum variance unbiased estimator is unique in the sense that if T_1 and T_2 are minimum variance unbiased estimators for $\Upsilon(\theta)$, then $T_1 = T_2$ almost surely. **(8, 7)**

3. (a) Define MVUE and MVBE. When does the inequality become equality in Cramer-Rao inequality? A random sample $X_1, X_2, ..., X_n$ is taken from a normal population with mean zero and variance σ^2. Examine if $\sum_{i=1}^{n} x_i^2/n$ is a MVB estimator for σ^2.

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(b) A random sample X_1, X_2, X_3, X_4 and X_5 of size 5 is drawn from a normal population with mean μ and variance σ^2. T_1, T_2 and T_3 are the estimatros used to estimate mean μ, where

$$T_1 = \frac{X_1+X_2+X_3+X_4+X_5}{5},\ T_2 = \frac{X_1+X_2}{2} + X_3 \text{ and } T_3 = \frac{2X_1+X_2+\lambda X_3}{3}.$$

Find λ such that T_3 is an unbiased estimator for μ. Are T_1 and T_2 unbiased estimators? State giving reasons, which is the best estimator among T_1, T_2 and T_3. **(8, 7)**

4. (a) Describe the method of minimum variance. Prove that MLE of the parameter α of a population having density function $f(x,\alpha) = \frac{2}{\alpha^2}(\alpha - x);\ \ 0 < x < \alpha$, for a sample of unit size is 2*x*, *x* being the sample value. Also, show that the estimate is biased.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\theta, 1)$ distribution. Obtain MVUE of parameter θ. **(8, 7)**

5. (a) Discuss the concept of interval estimation, confidence interval and confidence limits and provide suitable illustration. How they are constructed using t-distribution?

(b) Obtain 100(1 - α)% confidence limits (for large samples) for the parameter λ of the Poisson(λ) distribution. **(8, 7)**

6. (a) Explain the following terms:

(i) Type I and Type II errors
(ii) The best critical region
(iii) Power function of a test
(iv) Level of significance.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size *n* from normal $N(\theta, \sigma^2)$ distribution, where σ^2 is known. Obtain most powerful test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (< \theta_0)$. Also, find the power function of the test. **(8, 7)**

7. (a) Let p be the probability that a coin will fall head in a single toss in order to test $\mathrm{H_0{:}\,p} = \frac{1}{4}$ against $\mathrm{H_1{:}\,p} = \frac{5}{6}$. The coin is tossed 7 times and H_0 is rejected, if more than 5 heads are obtained. Find the probability of type I error and power of the test.

(b) Prove that, if w be uniformly most powerful test of size α for testing $H_0{:}\,\theta = \theta_0$ against $H_1{:}\,\theta \in \Theta_1$, then it is necessarily unbiased. **(8, 7)**

8. Explain the following terms **(Any three)** **(5x3=15)**

(i) Fisher-Neyman criterion.

(ii) Cramer-Rao inequality

(iii) Confidence intervals for large samples

(iv) Most powerful critical region and uniformly most powerful critical region.