

- (b) Show that the vector field $F(x,y) = (x+2y)i + (2x+y)j$ is conservative and find a scalar potential f for

F. Evaluate the line integral $\int_C F \cdot dR$, where C is any smooth path connecting $A(0,0)$ to $B(1,1)$.

- (c) Use Green's theorem to find the work done by the force field

$$F(x,y) = (3y-4x)i + (4x-y)j$$

When an object moves once counterclockwise around the ellipse $4x^2 + y^2 = 4$.

6. (a) Evaluate $\iint_S (x^2 + y^2) dS$, where S is the surface

of the hemisphere $z = \sqrt{1 - x^2 - y^2}$.

- (b) Let S be the portion the plane $x+y+z = 1$ that lies in the first octant, and let C be the boundary of S , traversed counterclockwise. Verify Stokes' theorem for the surface S and the vector field $F =$

$$F = -\frac{3}{2}y^2i - 2xyj + yzk.$$

- (c) Use the divergence theorem to evaluate.

$$\iiint_S F \cdot N dS$$

Where, $F = xyzi + xyzj + xyzk$, S is the surface of the box $0 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 3$, with outward unit normal vector N .

This question paper contains 4 printed pages]

Your Roll No. :

Sl. No. of Q. Paper : 1932 I

Unique Paper Code : 2352012402

Name of the Paper : Multivariate Calculus-DSC

Name of the Course : B.Sc. (H) Mathematics

Semester : IV

Time : 3 Hours

Maximum Marks : 90

Instructions for Candidates :

- Write your Roll No. on the top immediately on receipt of this question paper.
- All questions are compulsory and are of 15 marks each.
- Attempt any **two** parts from each Questions. Each part is of 7.5 marks.

1. (a) Consider the function:

$$f(x,y) = \begin{cases} \frac{2x^2 - x^2y^2 + 2y^2}{x^2 + y^2} & \text{for } (x,y) \neq (0,0) \\ A & \text{for } (x,y) = (0,0) \end{cases}$$

Find the value of A , which will make f continuous at the $(0,0)$.

- (b) The temperature at a point (x, y) on a given metal plate in the xy plane is determined according to the formula $T(x,y) = x^3 + 2xy^2 + y$ degrees. Compute the rate at which the temperature changes with distance if we start at $(2,1)$ and move
- parallel to the vector j
 - parallel to the vector i

- (c) A juice can is 12 cm tall and has a radius of 3 cm. A manufacturer is planning to reduce the height of the can by 0.2 cm and the radius by 0.3 cm. Use a total differential to estimate the percentage decrease in volume that occurs when the new cans are introduced. (Round to the nearest percent.)

2. (a) Let $w = f(t)$ be a differentiable function of t , where $t = (x^2 + y^2 + z^2)^{1/2}$. show that

$$\left(\frac{dw}{dt}\right)^2 = \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2$$

- (b) Find the directional derivative of $f(x, y) = e^{x^2} y^2$ at $P_0(1, -1)$ in the direction toward $Q(2, 3)$.
- (c) Find the absolute extrema of the function $f(x, y) = xy - 2x - 5y$ on the closed bounded set S , where S is the triangular region with vertices $(0, 0)$, $(7, 0)$ and $(7, 7)$.

3. (a) (i) Find the volume of the solid bounded below by the rectangle R in the xy -plane and above by the graph:

$$z = \frac{xy}{\sqrt{x^2 + y^2 + 1}}; \quad R: 0 \leq x \leq 1, 0 \leq y \leq 1$$

- (ii) Sketch the region of integration and compute the double integral:

$$\int_0^{\frac{\pi}{3}} \int_0^{y^2} \frac{1}{y} \sin \frac{x}{y} dx dy$$

- (b) (i) Compute double integral by converting to polar coordinates:

$$\int_0^3 \int_x^{\sqrt{9-x^2}} \cos(x^2 + y^2) dy dx$$

- (ii) Use a double integral to find the area bounded by $r = \cos 2\theta$.

- (c) (i) Evaluate Triple integral:

$$\iiint_D x dV$$

where, D is bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$.

- (ii) Compute the iterated triple integral.

$$\int_{-1}^2 \int_0^{\pi} \int_1^4 yz \cos xy dz dx dy$$

4. (a) Find the volume of the solid bounded by cylinders $y = z^2$ and $y = 2 - z^2$ and the planes $x = 1$ and $x = -2$
- (b) Evaluate triple integral

$$\iiint_D \frac{dx dy dz}{\sqrt{x^2 + y^2 + z^2}}$$

where, D is the solid sphere $x^2 + y^2 + z^2 \leq 3$

- (c) Use suitable change of variable find the area of the region R bounded by the hyperbolas $xy = 1$ and $xy = 4$ and the lines $y = x$ and $y = 4x$.

5. (a) Evaluate $\int_C F dR$, where $F = (y^2 - z^2)i + (2yz)j - x^2k$ and C is the curve defined parametrically by $x = t^2$, $y = 2t$ and $z = t$, for $0 \leq t \leq 1$.