[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3002 **L**

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : **B.A./B.Sc. (Programme) with Mathematics as Non-Major/ Minor DSC**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. The use of a calculator is not allowed.

1. (a) For each element a in a group, show that there is a unique element b in G such that $ab = ba = e$. Also find the inverse of *13* in *U(14)*. (7.5)

   (b) Show that $G = \{1, -1, \iota, -\iota\}$ forms a group under multiplication. (7.5)

   (c) Find the inverse of $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix} \in \mathrm{GL}(2, Z_{11})$. (7.5)

2. (a) Show that *{1,2,3}* under multiplication modulo *4* is not a group, but that *{1,2,3,4}* under multiplication modulo *5* is a group. (7.5)

   (b) State and prove the one-step subgroup test. (7.5)

   (c) Find the order of the group and the order of each element in the group when $G = U(10)$ and Z_{12}. (7.5)

3. (a) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$. (7.5)
   Compute $\alpha\beta, \beta\alpha$ and β^{-1}. Check whether α and β are even or odd permutations.

   (b) List the elements of the subgroups $\langle 20 \rangle$ and $\langle 10 \rangle$ in Z_{30}. Let a be a group element of order 30. List the elements of the subgroups $\langle a^{20} \rangle$, and $\langle a^{10} \rangle$. (7.5)

   (c) Find all of the left cosets of $\{1, 11\}$ in $U(30)$. (7.5)

4. (a) Define normal subgroup. Show that the alternating group A_n of even permutations is a normal subgroup of S_n. (7.5)

(b) Construct a complete Cayley table for D_4, the group of symmetries of a square. Find the inverse of each of the element of D_4. (7.5)

(c) Define group homomorphism. Show that the Kernel of homomorphism $\phi: \mathbb{R}^* \to \mathbb{R}^*$ given by $\phi(x) = |x|$ is a cyclic group. (7.5)

5. (a) Prove that intersection of any collection of subrings of a ring R is a subring of R. Does the result hold for the union of subrings? Justify. (7.5)

(b) Prove that finite Integral Domains are fields. Can the condition of finiteness be dropped? Justify. (7.5)

(c) (i) State and prove the subring test. (7.5)

(ii) Prove that the ring $\mathbb{Z}_p$ of integers modulo p where p is a prime, is an integral domain.

6. (a) Determine all the ring homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{30}$. (7.5)

(b) (i) Prove that the map $f(x) = x \bmod n$ from $\mathbb{Z}$ to $\mathbb{Z}_n$ is a ring homomorphism. (7.5)

(ii) Prove that if A be an ideal of a ring R and $1 \in A$, then $A = R$.

(c) Let R be the ring of all real-valued functions of a real variable. Prove that the subset S of all differentiable functions is a subring of R but not an ideal of R. (7.5)