[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 10156 **L**

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : **B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor-DSC**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. Each part carries **7.5** marks.
4. Use of Calculator not allowed.

1(a). Define greatest common divisor(gcd) and least common multiple(lcm) of two non-zero integers a and b and apply the definition to find gcd and lcm of the following:

(i) $2^2 \times 3^2 \times 5$,and $2 \times 3^3 \times 7^2$ (ii) $p^2 \times q^2 \times r^3$, and $p \times q^3 \times r^5$.

(b). Define a group. Show that the set $\mathbb{R}\backslash\{0\}$ forms a group under ordinary multiplication.

(c). Find the inverse of the element $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ in the general linear group $GL(2, \mathbb{Z}_7)$.

2(a). Let G be an abelian group with identity e. Prove that the set $H = \{x \in G: x^2 = e\}$ is a subgroup of G.

(b). Consider a finite group G and let $a \in G$. If $|a| = 18$, find the orders of elements $a^2, a^{12}, a^{21}, a^{23}, and\ a^{54}$.

(c). Form a Cayley's table for dihedral group D_3 of symmetries of an equilateral triangle. Is D_3 abelian?

3 (a) Let the permutation $\alpha = (13256)(23)(46512)$ compute the following:

(i) α^{-1}

(ii) Write α as product of disjoint cycles

(iii) Order of α

(b) Let H be a subgroup of G and let $a, b \in G$, then show that $aH = bH$ if and only if $a \in bH$.

(c) Let G be a finite group, and let $a \in G$ then show that $a^{|G|} = e$.

P.T.O.

4 (a) Let $\mathbb{Z}$ be the set integers is group under addition and $4\mathbb{Z} = \{0,\ \pm 4,\ \pm 8, . . .\}$ is normal subgroup of $\mathbb{Z}$. Construct all the left cosets of $4\mathbb{Z}$ in $\mathbb{Z}$ and construct the Cayley table.

(b) Prove that a factor group of a cyclic group is cyclic.

(c) Let $\mathbb{R}^*$be the group of nonzero real numbers under multiplication. Let $G = GL(2, \mathbb{R})$ and mapping $\phi: G \to \mathbb{R}^*$defined by $\phi(A) = \det A$. Prove that mapping ϕ is a homomorphism and also determine the $Ker(\phi)$.

5. **(a)** Prove that the intersection of any family of ideals of a ring R is an ideal of R.

(b) Show that the set $\mathbb{Q}\sqrt(3) = \{a + b\sqrt{3} \mid a, b \in \mathbb{Q}\}$ is a field.

(c) Define the characteristic of a ring. Prove that the characteristic of a field is either zero or a prime number.

6. **(a)** State the ideal test. Prove that $4\mathbb{Z} = \{4n \mid n \in \mathbb{Z}\}$ is an ideal of $\mathbb{Z}$.

(b) Show that the mapping $\phi: \mathbb{Z}_6 \to \mathbb{Z}_{12}$ defined by $\phi(x) = 2x$. is a ring homomorphism.

(c) Let $\phi: R \to$S be a ring homomorphism and let A be an ideal of R. Prove that $\phi(A) = \{\phi(a) \mid a \in A\}$ is an ideal of $\phi(R)$.