(c) Let $<f_n>$ be a sequence of continuous functions that converges pointwise to a function f on [a, b].

(i) Does it mean that f is continuous on [a, b]? Justify your answer.

(ii) State the condition on the convergence of $<f_n>$ that ensures f is continuous on [a,b] 7.5

6. (a) State the Weierstrass M-Test for uniform convergence of series of functions. Apply this test to the following series,

$$\sum_{k=1}^{\infty} x^k e^{-kx} \text{ on } (0,\ 1)$$ 7.5

(b) Find the interval of convergence and the radius of convergence for the following power series. 7.5

(i) $\sum_{k=0}^{\infty} \frac{k!}{2^k} x^k$ (ii) $\sum_{k=1}^{\infty} \left(\frac{k+1}{k}\right)^k x^k$.

(c) Show that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots,$ $-1 < x \le 1$ and using Abel's theorem, deduce that

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$$ 7.5

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This question paper contains 4 printed pages]

Your Roll No. :......................

Sl. No. of Q. Paper :**10175** **I**

Unique Paper Code :2352203602

Name of the Paper :DSC-Elementary Mathematical Analysis

Name of the Course :**NEP-UGCF B.A. (Prog) With Mathematics as Major**

Semester : VI

Time : 3 Hours ***Maximum Marks : 90***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each question.

(c) **All** questions are compulsory.

1. (a) Define the limit of a function in terms ϵ-δ. Use this definition to show that 7.5

$$\lim_{x \to 2} (2x+5) = 9$$

(b) Use the Sequential Criterion for limits to prove that $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist in $\mathbb{R}$. 7.5

(c) State and prove the sequential criterion for continuity of a real-valued function f at $c \in \mathbb{R}$. 7.5

2. (a) Define the continuity of a function at a point in terms of $\epsilon-\delta$. Use $\epsilon-\delta$ definition, to show that $f(x) = x^2-2x-1$ is continuous at $x_0 = 3$. 7.5

(b) State the Location of Root Theorem. Also, prove that $\cos(x) = x$ for some x in $\left(0, \frac{\pi}{2}\right)$. 7.5

(c) Define uniform continuity of a function f on interval 1. Use it to prove that the function $f(x) = \frac{1}{x}$ is uniformly continuous on the interval $[1, \infty)$. 7.5

3. (a) Define Riemann Integrability of a bounded function f on [a,b]. For any partition P of [a,b], prove that $\underline{S}(f,P) \leq \overline{S}(f,P)$. 7.5

(b) Let f(x) = 5 −3x be defined on [0,2]. For the partitiion $P = \left\{0, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 2\right\}$ of [0,2], find $\underline{S}(f,P)$ and $\overline{S}(f,P)$. 7.5

(c) Show that $f: \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 1, \text{ if } x \text{ is rational} \\ -1, \text{ if } x \text{ is irrational} \end{cases}$$

is not integrable over [0,1]. 7.5

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4. (a) Use sequential criteria for integrability to show that the function $f(x) = x^2$ is integrable on [0,1] and find $\int_0^1 f$. 7.5

(b) Porve that f(x)= 2 is integrable on [3,5]. Also, find $\int_3^5 f$. 7.5

(c) let $f, g: [a,b] \to \mathbb{R}$ be integrable functions on [a,b]. 7.5

(i) If $f(x) \geq 0$, for all $x \in [a,b]$ then prove that $0 \leq \int_a^b f$.

(ii) If $f(x) \leq g(x)$, for all $x \in [a,b]$, then prove that $\int_a^b f \leq \int_a^b g$.

5. (a) Discuss the pointwise convergence of the following sequences of functions on [0,1]. 7.5

(i) $<f_n>$ where $f_n(x) = x^n$,

(ii) $<g_n>$ where $g_n(x) = \frac{x^2+n}{n}$

(b) Define uniform convergence for sequence of functions. Show that the sequence of functions $<f_n>$ is uniformly convergent on [0,a] for a>0, where

$$f_n(x) = \frac{x}{x+n}, x \in [0,a].$$ 7.5

P.T.O.