Roll No........................

Serial No. 5197

Name of Course	**: B.Sc. Hons.-(Physics) NEP: UGCF-**
2022 Semester	**: VIII- Semester (GE Paper)**
Name of the Paper	**: Quantum Mechanics**
Unique Paper Code	**: 2224002006**
Duration	**: 3 Hours Maximum Marks : 90**

Instructions for students:

Write your Roll No. on the top immediately on receipt of this question paper.

First Question is compulsory. Attempt five questions in total.

All questions carry equal marks.

Use of non-programmable scientific calculator is permitted.

Q1. Attempt any six of the following sub questions, each carrying three marks.

(a) Define a Hermitian operator and state why physical observables must be represented by them.

(b) Check if the operators $\hat{x}^2$ and $\hat{p}_x$ commute with each other by evaluating $[\hat{x}^2, \hat{p}_x]$.

(c) State the physical conditions that a wave function $\psi(x)$ must satisfy to be considered acceptable.

(d) Why is it impossible for a particle trapped in a one-dimensional box to have a zero ground-state energy?

(e) Express the angular momentum operator component $\hat{L}_z$ in spherical polar coordinates.

(f) A wave function is given by $\psi(x) = A\sin\left(\frac{\pi x}{L}\right)$ for $0 \leq x \leq L$. Determine the normalization constant A.

(g) Briefly explain the concept of phase velocity and state its mathematical relationship to group velocity.

Q2. (a) Formulate the probability current density $\vec{J}$ for a three-dimensional wave function. Prove the continuity equation $\frac{\partial\rho}{\partial t} + \nabla \cdot \vec{J} = 0$ and interpret its physical meaning regarding probability conservation. **(10 Marks)**

(b) Show that if two operators $\hat{A}$ and $\hat{B}$ commute, they share a common set of simultaneous eigenfunctions. **(8 Marks)**

Q3. (a) Consider a particle of mass m trapped inside a symmetric one-dimensional infinite potential well defined by:

$$V(x) = \begin{cases} 0, & -a < x < a \\ \infty, & \text{otherwise} \end{cases}$$

Solve the time-independent Schrödinger equation to derive the normalized energy eigenfunctions and show that the states alternate in parity. **(10 Marks)**

(b) Find the energy eigenvalues for the ground state and the first excited state of a particle in this symmetric well if the total width of the well ($2a$) is equal to 2 Å. **(8 Marks)**

Q4. (a) Using the power series method (Frobenius method), solve the time-independent Schrödinger equation for a 1-D linear harmonic oscillator to find its energy eigenvalues. **(10 Marks)**

(b) Given the raising operator $a^\dagger$ and lowering operator a, prove the following operational relationships:

1. $a|n\rangle = \sqrt{n}\,|n-1\rangle$
2. $a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$ **(8 Marks)**

Q5. (a) For a particle in the ground state ($n = 0$) of a 1-D linear harmonic oscillator, evaluate the expectation values $\langle x\rangle$ and $\langle p_x\rangle$. Use these to determine the uncertainties Δx and Δp_x. **(10 Marks)**

(b) A quantum harmonic oscillator has an angular frequency of $\omega = 3 \times 10^{14}$ rad/s. Calculate its zero-point energy in electron-volts (eV). **(8 Marks)**

Q6. (a) Separate the three-dimensional time-independent Schrödinger equation in spherical polar coordinates (r, θ, ϕ) for a central potential $V(r)$. Explicitly isolate and solve the azimuthal (ϕ) dependent differential equation. **(10 Marks)**

(b) A particle is represented by an initial Gaussian wave packet $\psi(x,0) = \left(\frac{1}{\pi a^2}\right)^{1/4} \exp\left(-\frac{x^2}{2a^2}\right)$. Describe qualitatively how and why this wave packet spreads as time progresses ($t > 0$). **(8 Marks)**