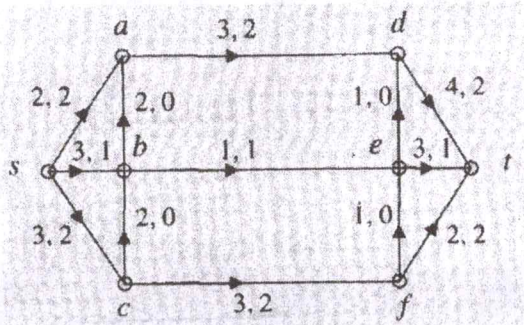
(b) In the given directed network show that the given flow is not maximum by finding a flow-augmenting chain from s to t. What is the slack in your chain?



(c) Five ladies have men friends as shown in the following table.

Lady	Men Friends
Karen	David, Stuart, Paul, Roger
Mary	Stuart, Philip
Aurie	David, Paul
Pamela	Paul, Roger
Lynne	Stuart, Paul

Draw a bipartite graph depicting this situation. Find a way in which each lady can marry a man she knows, or use Hall's Marriage Theorem to explain why no such matching exists.

***

This question paper contains 8 printed pages]

Your Roll No. : .....

Sl. No. of Q. Paper : **10217** **I**

Unique Paper Code : 2352202402

Name of the Paper : Introduction of Graph Theory

Name of the Course : **Bachelor of Arts (Programme) with Mathematics as Major (DSC-2)**

Semester : IV

Time : 3 Hours ***Maximum Marks : 90***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **all** the questions by selecting any two parts out of the three **parts (a), (b)** and **(c)** for each of the **six** questions.

(c) **All** the questions carry equal marks.

(d) Use of Calculator is not allowed.

1. (a) Define degree sequence in a graph and draw the following graphs. Also, determine how many edges each graph has:

(i) K_4 (ii) $K_{3,2}$

(iii) $K_{1,5}$

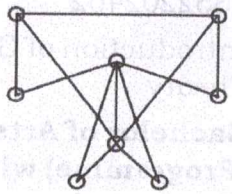
(b) Explain with an example, the difference between an

(i) Eulerian circuit and a Hamiltonian cycle.

(ii) Eulerian trail and a Hamiltonian path.

P.T.O.

(c) Is the graph given below Hamiltonian? Is it Eulerian? Explain your answers.



2. (a) Given **two** graphs:

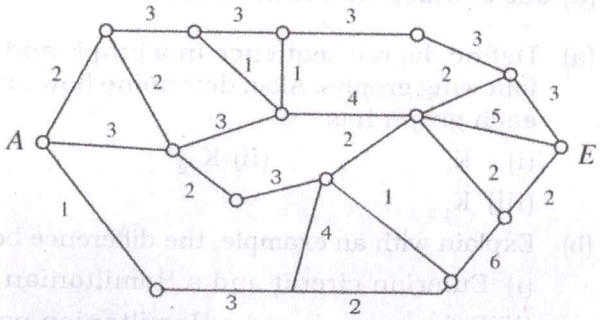
G_1: Vertices = {A, B, C, D}, Edges= (AB, BC, CD, DA};

G_2 : Vertices {W, X, Y, Z}, Edges = {WY, XY, YZ, ZW}.

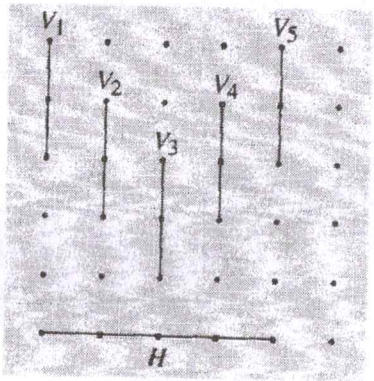
Are the graphs G_1 and G_2 isomorphic? Justify your answer.

(b) Explain how the Königsberg Bridge problem is directly related to the study of Eulerian graphs.

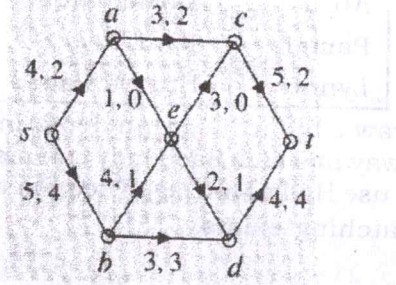
(c) Use the improved version of Dijkstra's algorithm to find the length of a shortest path from vertex A to vertex E in the given weighted graph. Label all the vertices.



(c) Draw the line of sight graph G associated with the given net pattern. Determine the chromatic number $\chi(G)$ and find a corresponding partition of nets.

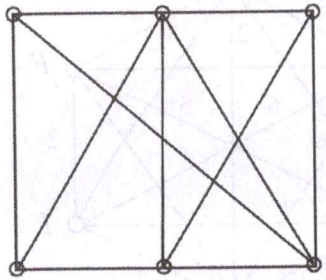


6. (a) For the network given below verify the law of conservation of flow at vertices a, e and d. Find the value of the integrated flow. Is the given flow maximum? Explain. Can the flow be increased along the path sbedt? If so, by how much?



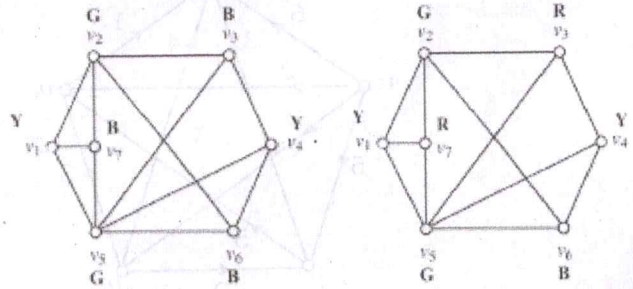
P.T.O.

5. (a) Define a planar graph and a plane graph. Show that the following graph is planar by drawing an isomorphic plane graph with straight edges.

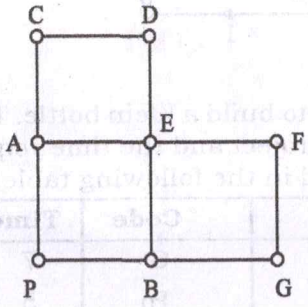


Also label the region defined by your plane graph and list the edges that form the boundary of each region.

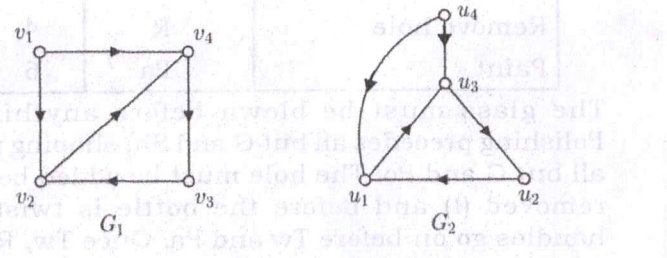
(b) Define the chromatic number of a graph. Determine the chromatic number of the following graphs.



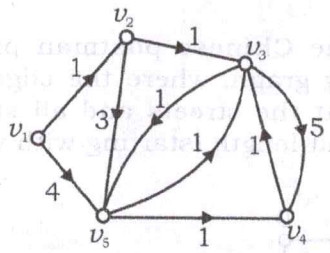
3. (a) Solve the Chinese postman problem for the following graph, where the edges of the graph represent the streets and all streets have the same unit length (starting with vertex P).



(b) When are digraphs said to be isomorphic? Are the following digraphs G_1 and G_2 isomorphic? Justify.



(c) Apply Bellman-Ford algorithm to find the shortest distances from v_1 to all other vertices in the given weighted digraph.



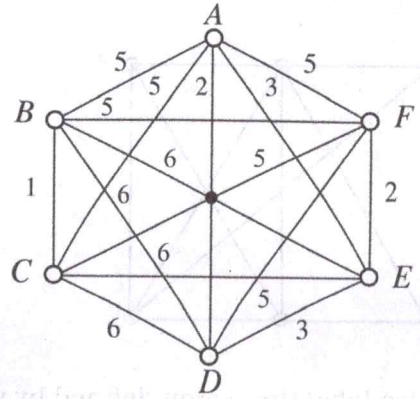
4. (a) Suppose we want to build a Klein bottle. The tasks involved in this project and the time required for each are recorded in the following table.

Task	Code	Time
Blow Glass	G	7
Polish	Po	5
Shape	Sh	9
Add hole	A	4
Affix handles	H	5
Twist	Tw	2
Remove hole	R	4
Paint	Pa	6

The glass must be blown before anything else. Polishing precedes all but G and Sh; shaping precedes all but G and Po. The hole must be added before it is removed (!) and before the bottle is twisted. The handles go on before Tw and Pa. Once Tw, R, and Pa are completed, the project is finished.

What type of scheduling problem is this (type I or II)? Draw the appropriate directed network

(b) Use Kruskal's algorithm to find the minimum spanning tree for the following graph.



(c) Apply Bellman's algorithm to the digraph below to find the length of the shortest path from v_0 to every other vertex.