[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 5152 **L**

Unique Paper Code : 2354000019

Name of the Paper : Optimization Techniques

Name of the Course : **Common Prog Group-NEP-UGCF 2022**

Semester : VIII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **one** part from each question.

1. (a) A firm having two sources S_1 and S_2 wishes to ship its products to two destinations D_1 and D_2. The number of units available at S_1 and S_2 are 5 and 25 respectively and the product demands at D_1 and D_2 are 20 and 10 respectively. The firm, instead of shipping directly from sources to destinations, decides to investigate the possibility of transshipment. The unit transportation costs (in rupees) are given in the following table:

		Sources		Destinations		
		S_1	S_2	D_1	D_2	Available
Sources	S_1	0	2	3	4	5
	S_2	2	0	2	4	25
Destinations	D_1	3	2	0	1	----
	D_2	4	4	1	0	----
Demand		----	-----	20	10	

(i) Convert the given problem into transportation problem. Also, make a node diagram for the given problem.

P.T.O.

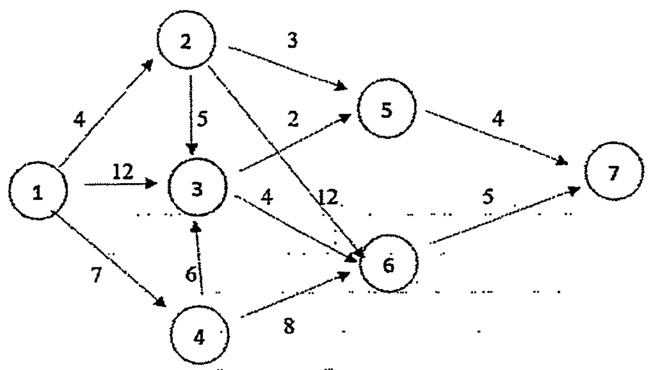
(ii)Find the initial basic feasible solution by North West Corner Rule.

(b) In a cargo loading problem, there are 4 items of different weights/unit and different value/unit which are given as below:

Item (i)	w_i (kg/unit)	p_i (Rs/ unit)
1	1	1
2	3	5
3	4	7
4	6	11

The maximum cargo load is restricted to 17. How many units of each item be loaded to maximize the value?

2. (a) Suppose we want to select the shortest highway route between two cities. The network in given figure provides the possible routes between the starting city node '1'and the destination city at node '7'. Solve by the method of backward recursive equation and use it to find the optimal solution for the following data:



(b) Consider the following transshipment problem with two sources and two destinations. The costs for shipment in rupees are given in the following table. Determine an optimum shipping schedule.

		Source		Destination		
		S1	S2	D1	D2	Supply
Source	S1	0	1	6	2	4
	S2	1	0	8	2	5
Destination	D1	0	7	0	3	-
	D2	1	0	3	0	-
Demand	-	-	-	3	6	

3. (a) Solve the following ILPP by using Gomory's Cutting Plane Method

$$\text{Max } Z = x_1 + x_2$$
$$\text{Subject to } 3x_1 + 2x_2 \leq 12$$
$$x_2 \leq 2$$
$$x_1, x_2 \geq 0 \text{ and both are integers.}$$

(b) Consider the following ILLP

$$\text{Max } Z = 4x_1 + 5x_2$$
$$\text{Subject to } x_1 + 4x_2 \leq 10$$
$$53x_1 - 4x_2 \leq 6$$
$$x_1, x_2 \geq 0 \text{ and both are integers}$$

(i) Convert the above LPP into the standard Linear programming problem.

(ii) Write the two LPP's by branching from x_2 and find the next branching variable, if the initial solution is $x_1 = 4$ and $x_2 = 1.5$, the maximum value of $Z = 23.5$, with the last simplex table as follows:

c_j			4	5	0	0
c_B	B	x_B	x_1	x_2	s_1	s_2
5	x_2	3/2	0	1	3/16	-1/16
4	x_1	4	1	0	1/4	1/4
	$z_j - c_j$		0	0	-31/16	-11/16

4. (a) Consider the following ILPP

$$\text{Max } Z = 4x_1 + 3x_2$$
$$\text{Subject to } 3x_1 + 4x_2 \leq 12$$
$$64x_1 + 2x_2 \leq 9$$
$$x_1, x_2 \geq 0 \text{ and both are integers.}$$

(i) Find the optimal solution of the defined problem by using the simplex method.

(ii) Solve the given ILPP graphically.

(b) Consider the following optimal simplex tableau of the associated Linear Programming Problem for a given AILP (with maximization from and x_3, x_4 as slack variables) with Max $Z = 53/10$. Generate the Gomory's cut through x_2 and hence solve the given AILP.

c_j			1	1	0	0
c_B	B	x_B	x_1	x_2	x_3	x_4
1	x_2	9/5	0	1	3/10	-1/10
1	x_1	7/2	1	0	-1/4	1/4
	$z_j - c_j$		0	0	1/20	3/20

5. (a) (i) Define a convex function on a convex set S $\subseteq R^n$.
Is the function $f(x) = -|x|, x \in R$, convex? Justify your answer using definition.
(ii) Suppose that $f(x)$ and $g(x)$ are concave functions defined on a convex set S $\subseteq R^n$. Show that the sum of these two concave functions, $(f + g)(x)$ is also a concave function on S $\subseteq R^n$.

(b) Examine the convexity/strict convexity for the following functions:
(i) $f(x_1, x_2) = 2x_1^2 + x_2^2 + 4x_1x_2$
(ii) $f(x_1, x_2) = 4x_1^2 + x_2^2 + 4x_1x_2$

6. (a) (i) Define a quasi-convex function on a convex set S $\subseteq R^n$. Give an example of a quasi-convex function which fails to be continuous in its domain.
(ii) Define a pseudolinear function on a convex set S $\subseteq R^n$. Give an example of a pseudolinear function.

(b) (i) Show that sum of two pseudo-convex functions need not be a pseudo-convex function on R.
(ii)Solve the following linear fractional programming problem by the simplex algorithm:

$$Max\ Z = \frac{x_1}{x_1+x_2+1}$$

subject to
$$x_1 + x_2 \leq 1$$
$$x_1 + 2x_2 \leq 1$$
$$x_1, x_2 \geq 0$$