(b) Prove that Taylor series of

$$e^z = e\sum_{n=0}^{\infty} \frac{(z-1)^n}{n!} \left(|z-1| \leq \infty\right).$$

(c) Find the residue at z = 0, of the functions:

(i) $\dfrac{z-\sin z}{z}$

(ii) $\dfrac{1}{z+z^2}$

6. (a) Define an isolated singular point. For the below given functions, find the isolated singular point and determine whether it is a removable singular point, an essential singular point, or a pole:

(i) $\dfrac{z^2}{1+z}$

(ii) $\dfrac{\cos(z)}{z}$

(b) Prove that

$$\operatorname{Res}_{z=-1} \frac{z^{1/4}}{z+1} = \frac{1+i}{\sqrt{2}}, \qquad \left(|z| > 0, 0 < \arg z < 2\pi\right).$$

(c) Find the value of the integral

$$\int_C \frac{3z^3+2}{(z-1)(z^2+9)} dz,$$

taken counterclockwise around the circle $|z| = 4$.

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This question paper contains 4 printed pages]

Your Roll No.	:	....
Sl. No. of Q. Paper	:	**5151** **I**
Unique Paper Code	:	2354000018
Name of the Paper	:	Elements of Complex Analysis-GE
Name of the Course	:	**COMMON PROG GROUP**
Semester	:	VIII

Time : 3 Hours ***Maximum Marks : 90***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **all** questions by selecting **two** parts from each question.

(c) Each question carries **15** marks each.

(d) Use of Calculator is **not** allowed.

1. (a) Let f be a function defined at all the points z in some deleted neighborhood of a point z_0 and $w_0 \in \mathbb{C}$. Define $\lim_{z \to z_0} f(z) = w_0$. Further, if $f(z) = \left(\dfrac{z}{\bar{z}}\right)^2$, then show that $\lim_{z \to 0} f(z)$ does not exist.

(b) let $f:\mathbb{C}\to\mathbb{C}$ be a function that is continuous and non-zero at $z_0\in\mathbb{C}$. Show that there exists a neighbourhood of z_0 in which $f(z)\neq 0$ for all z in that neighbourhood.

(c) Let $f:\mathbb{C}\to\mathbb{R}$ be a function such that $f(z)=\mathrm{Re}(z), Z\in\mathbb{C}$. Show that f'(z) does not exist at any point z.

2. (a) Using the Cauchy-Riemann equations, determine for the function $f(z)=\frac{1}{z}$ where f'(z) exists and find its value.

(b) Define an entire function. Further, show that f(z) = sinx coshy+icosx sinhy, where z = x+iy, is an entire function. Also, show that f''(z) = –f(z).

(c) Define the sine and cosine functions of a complex variable z. Further, show that $\sin(z_1+z_2)=\sin z_1\cos z_2+\cos z_1\sin z_2$

3. (a) if w(t) = u(t)+iv(t) is a complex-valued function of a real variable t and w'(t) exists, prove that $\frac{d}{dt}[w(t)]^2=2w(t)w'(t)$.

(b) Use parametric representation for C to evaluate $\int_C \frac{z+2}{z}dz$, where C is the semicricle $z=2e^{i\theta}$ $(0\leq\theta\leq 2\pi)$

(c) Without evaluating the integral, show that $\left|\int_C \frac{dz}{z^2-1}\right|\leq\frac{\pi}{3}$, when C is the arc of the circle $|z|=2$ from $z=2$ to $z=2i$ that lies in the first quadrant.

4. (a) By finding an antiderivative, evaluate the following integrals, where the path is any contour between the indicated limits of integration

(i) $\int_i^{i/2} e^{\pi z}dz$

(ii) $\int_1^3 (z-2)^3\,dz$

(b) State the Cauchy-Goursat theorem and hence apply it to evaluate $\int_c f(z)dz$, where the contour C is the positively oriented unit circle $|z|=1$ and

(i) $f(z)=\frac{1}{z+2}$

(ii) $f(z)=\sin(z^2)$

(c) Let C the positively oriented circle |z| = 1 about the origin. Evaluate

(i) $\int_C \frac{\cos z}{z(z^2+9)}dz$

(ii) $\int_C \frac{\exp(2z)}{z^4}dz$

5. (a) Find the Maclaurin scries for the function f(z) = sin (z).