[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1756

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Unique Paper Code : 2372013602

Name of the Paper : Design of Experiments (NEP-UGCF)

Name of the Course : **B.Sc. (H) Statistics**

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question **1** is compulsory.
3. Select **four** questions from the remaining **six** questions.
4. Use of non-programmable scientific calculator is allowed.

1. Attempt any 6 parts:

(a) A student of Statistics has conducted a complete randomized design with different treatments to complete her experiment. The results of this experiment are summarized in the following ANOVA display:

Source	df	Sum of Squares	Mean squares	F
Treatment	--	--	12	--
Error	--	36	--	
Total	15	108		

Answer the following questions about this experiment:

(i) The sum of squares for the treatment is _____

(ii) The number of degrees of freedom for treatments in the experiment is_____

(iii) The number of degrees of freedom for error is _____

P.T.O.

(iv) The mean sum of squares due to error is _____

(v) The value of the test statistic is_____

(b) Derive the expression to measure the efficiency of RBD over CRD, stating clearly the assumption used in the derivation.

(c) An agricultural scientist wants to study the effect of three varieties of wheat V_1, V_2, and V_3 on crop yield. A field is divided into 12 plots. The varieties are assigned to the plots at random, each variety being grown on four plots. Since soil fertility varies across the field, the field is divided into four blocks of equal size.

Answer the following questions about this experiment:

(i) Name of the design and the number of treatments.

(ii) Number of experimental units.

(iii) Number of replications.

(iv) The principles of Design of Experiments used in this experiment.

(d) Given a BIBD with parameters $v = b = 7, r = k = 3, \lambda = 1$.

1	2	3	4	5	6	7
2	3	4	5	6	7	1
4	5	6	7	1	2	3

Obtain the dual of the given design along with the parameters.

(e) Define a treatment contrast. When are two contrasts said to be orthogonal? Let $L_A = [a - (1) + b - ab]$, $L_B = [(1) - a - b + ab]$ and $L_C = [a + b - ab - (1)]$. Are these contrasts? If yes, which of them are orthogonal to each other?

(f) The key block in a 2^4 Factorial experiment is given as

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Identify all the confounded effects. Write down the contents of the remaining blocks.

(g) Consider the following fractional factorial designs:

D_1: 2^{6-2} with E = ABCD, F = ABD

D_2: 2^{6-2} with E = ABC, F = BCD

What is the resolution of these designs? which design would you prefer and why?

(6×5=30)

2. (a) Discuss the three principles of the design of experiments. How are these principles used in a Latin Square design?

(b) Describe the complete statistical analysis of an LSD to test the hypothesis of equality of all treatment effects. Also, write the complete ANOVA table.

(6. 9)

3. (a) Prove that in an RBD, the mean sum of squares due to error is always an unbiased estimate of the error variance while the mean sum of squares due to treatments is a biased estimator. Under what condition is it unbiased?

(b) Derive the least square estimates of the parameters in a randomized block design, clearly stating the model. Also, find the variance of the estimated parameters.

(6, 9)

4. (a) Define resolvable and affine resolvable BIBDs. Also, prove that for a resolvable BIBD with parameters (v, b, r, k, λ); $b \geq v + r - 1$.

(b) Define a Balanced Incomplete Block Design. Show that a BIBD with parameters (v, b, r, k, λ) satisfies the following relationships:

i) $b \geq v$

ii) $b \geq v + r - k$

(6, 9)

5. (a) Explain Yates technique for calculating sum of squares due to main effects and interaction effects in case of 3^2 factorial experiment with 4 replications.

(b) Construct a 2^5 factorial experiment with 5 factors A, B, C, D and E in 2^2 blocks in two replicates confounding BCD and ADE in one replicate and CE and ABE in other replicate. Identify if any other effect is also confounded in each replicate. Also write down only the first two columns of the ANOVA table.

(6, 9)

6. (a) An experiment for 5 treatment factors. each with two levels. is designed such that only a single replicate of the 32 treatment combinations is run. Suggest a suitable method for analyzing such a design.

(b) What is confounding? Construct a completely confounded 2^3 factorial experiment in blocks of size 4 with 8 blocks, so that it completely confounds the second order interaction. How the arrangement can be modified so as to recover at least partial information about the 2 and 3 factor interaction components and complete information about all the main effects. (6, 9)

7. (a) Define fractional factorial designs. Who first proposed the idea of fractional replications in large factorial experiments? Explain two differences between fractional factorial experiments and confounded designs.

(b) Suppose a 2^5 experiment was to be run as a CRD, but the researcher could only run it with 8 experimental units. The following treatment combinations were chosen as a fraction to run the experiment:

a, ade, b, bde, ce, cd, abce, abcd

i) Identify the design and the defining relation for the design,

ii) How many alternate fractions can you generate for this design?

iii) Find Resolution of this design,

iv) Write down the complete aliasing scheme for this design.

(6, 9)

(1000)