[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2925 **L**

Unique Paper Code : 2352203601

Name of the Paper : DSC – Probability and Statistics

Name of the Course : **Bachelor of Arts / Bachelor of Science (Programme) with Mathematics as non-major/minor**

Semester : VI

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.
5. Use of non-programmable scientific calculator and statistical tables is permitted.

1. (a) Construct a stem and leaf display of the following data:

5.9	7.2	7.3	6.3	8.1	6.8	7.0	7.6	6.8	7.7
6.5	7.0	6.3	7.9	9.0	8.2	8.7	7.8	9.7	7.8
7.4	7.7	9.7	11.8	10.7	11.6	11.3			

Also, comment on interesting features of the display.

P.T.O.

(b) Consider the following data on endotoxin concentration in settled dust both for a sample of urban homes and for a sample of farm homes:

U	6.0	5.0	11.0	33.0	4.0	5.0	80.0	18.0	35.0	17.0	23.0
F	4.0	14.0	11.0	9.0	9.0	8.0	4.0	20.0	5.0	8.9	21.0
	9.2	3.0	2.0	0.3							

(i) Determine the value of the sample standard deviation for each sample, interpret these values and then contrast variability in the two samples.

(ii) Compute the fourth spread for each sample and compare.

(c) Zinfandel is a popular red wine varietal produced almost exclusively in California. It is rather controversial among wine connoisseurs because its alcohol content varies quietly substantially from one producer to other. In May 2003, the author went to the website klwines.com, randomly selected 10 zinfandels from among the 325 available, and obtained the following values of alcohol content (%) :

14.8	14.5	16.1	14.2	15.9	13.7	16.2	14.6	13.8	15.0

Calculate mean, median and sample variance.

2. (a) Let X be a Bernoulli random variable with pmf $p(1) = p$ and $p(0) = 1 - p$.

(i) Compute $E(X^2)$.

(ii) Show that $V(X) = p(1 - p)$.

(iii) Compute $E(X^{79})$.

(b) A friend of mine is giving a dinner party. His current wine supply includes 8 bottles of zinfandel, 10 of merlot, and 12 of cabernet (he only drinks red wine), all from different wineries.

(i) If 6 bottles of wines are to be randomly selected from the 30 for serving, how many ways are there to do this.

(ii) If 6 bottles are randomly selected, what is the probability that all of them are the same variety.

(c) For any three events A, B and C prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

3. (a) Six boxes of components are ready to be shipped by a certain supplier. The number of defective components in each box is as follows :

Box	1	2	3	4	5	6
No. of Defectives	0	2	0	1	2	0

One of these boxes is to be randomly selected for shipment to a particular customer. Let X be the number of defectives in the selected box. Find pmf of X.

(b) Let X be the number of students who show up for a professor's office hour on a particular day. Suppose that the pmf of X is $p(0) = .20$, $p(1) = .25$, $p(2) = .30$, $p(3) = .15$, and $p(4) = .10$.

(i) Draw the corresponding probability histogram.

(ii) What is the probability that at least two students show up? More than two students show up?

(iii) What is the probability that between one and three students, inclusive, show up?

(iv) What is the probability that the professor shows up?

(c) A certain brand of upright freezer is available in three different rated capacities: 16 ft^3, 18 ft^3 , and 20 ft^3. Let X = the rated capacity of a freezer of this brand sold at a certain store. Suppose that X has pmf

x	16	18	20
p(x)	.2	.5	.3

(i) Compute E(X), $E(X^2)$, and V(X).

(ii) If the price of a freezer having capacity X is $70X - 650$, what is the expected price paid by the next customer to buy a freezer?

(iii) What is the variance of the price paid by the next customer?

4. (a) Each of six randomly selected cola drinkers is given a glass containing cola S and one containing cola F. The glasses are identical in appearance except for a code on the bottom to identify the cola. Suppose there is actually no tendency among cola drinkers to prefer one cola to the other. Then p = P (a selected individual prefers S) = .5, so with X = the number among the six who prefer S.

(i) What is the probability that exactly three prefer S?

(ii) What is the probability that at least three prefer S?

(iii) What is the probability that at most one prefers S?

(b) Compute the following binomial probabilities directly from the formula b(x; n, p).

(i) b(3; 8, .35)

(ii) P(3 ≤ X ≤ 5) when n = 7 and p = .6

(iii) P(1 ≤ X) when n = 9 and p = .1

(c) The current in a certain circuit as measured by an ammeter is a continuous random variable X with the following density function:

$$f(x) = \begin{cases} .075x + .2 & 3 \le x \le 5 \\ 0 & otherwise \end{cases}$$

(i) Verify that the total area under the density curve is indeed 1.

(ii) Calculate $P(X \leq 4)$.

(iii) Calculate $P(3.5 \leq X \leq 4.5)$.

5. (a) The Department of Statistics has a lab with six computers reserved for statistics majors. Let X denote the number of these computers that are in use at a particular time of day. Suppose that the probability distribution of X is as given in the following table; the first row of the table lists the possible X values and the second row gives the probability of each such value.

x	0	1	2	3	4	5	6
$p(x)$	0.05	0.10	0.15	0.25	0.20	0.15	0.10

Find the probability that (a) at most 2 computers are in use, (b) at least 3 computers are in use, (c) between 2 and 5 computers inclusive are in use, (d) the number of computers in use is strictly between 2 and 5.

(b) Suppose pulses arrive at a counter at an average rate of six per minute. Find the probability that in a 0.5-min interval at least one pulse is received.

(c) The weight distribution of parcels sent in a certain manner is normal with mean value 12 lb and standard deviation 3.5 lb. The parcel service wishes to establish a weight value c beyond which there will be a surcharge. What value of c is such that 99% of all parcels are under the surcharge weight?

6. (a) The distribution of egg weights (g) of a certain type is normal with mean value 53 and standard deviation 0.3. Let $X_1, X_2, ..., X_{12}$ denote the weights of a dozen randomly selected eggs; these X_i's constitute a random sample of size 12 from the specified normal distribution. Find the mean and standard deviation of the sample total weight T_0. And, determine the probability that the sample total weight T_0 is between 635 and 640.

(b) Consider the following data on y = porosity (%) is related to x = unit weight (pcf) in concrete specimens.

x	110	112	112	113	113	115	115	120
y	20	17	19	16	17	13	14	11

Find the equation of regression line. And estimate the porosity (y) when unit weight (x) is 100.

(c) Consider the following data of on x = rainfall volume (m^3) and y = runoff volume (m^3) for a particular location.

x	5	12	14	17	23	30	40	47	55	67
y	4	10	13	15	15	25	27	46	38	46

Compute the coefficient of correlation between x and y. And, how would you describe the nature of relationship between the two variables.