

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2285

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Unique Paper Code : 2352203601

Name of the Paper : DSC – Probability and Statistics

Name of the Course : **Bachelor of Arts / Bachelor of Science (Programme)
with Mathematics as non- major / minor**

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.
5. Use of non-programmable scientific calculator and statistical tables is permitted.

1. (a) Construct stem and leaf display of the following data and comment on any interesting features.

136	140	155	150	144	147	169	152	161	139
142	145	149	155	158	160	150	155	160	

P.T.O.

- (b) The May 1, 2009, issue of The Mont Clarian reported the following home sale amounts for a sample of homes in Alameda, C A that were sold the previous month (1000s of \$) :

590	815	575	608	350	1285	408	540	555	679
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- (i) Calculate and interpret the sample mean and median.
- (ii) Suppose the 6th observation had been 985 rather than 1285. How would the mean and median change?
- (c) An article reported the following data on oxidation-induction time (min) for various commercial oils :

87	103	130	160	180	195	132	145	211	105
145	153	152	138	87	99	93	119	129	

Calculate the sample variance and standard deviation. Answer without reperforming the calculations, what will be the new sample variance and deviation if each observation is multiplied by 4.

2. (a) An individual has three different email accounts. Most of her messages, in fact 70%, come into account #1, whereas 20% come into account #2 and the remaining 10% into accounts #3. Of the messages into account #1, only 1% are spam, whereas the corresponding percentage for accounts #2 and #3 are 2% and 5%, respectively. What is the probability that a randomly selected message is spam?

- (b) A new magazine publishes three columns entitled “Art” (A), “Books” (B) and “Cinema” (C). Reading habits of a randomly selected readers with respect to these columns are as follows:

Reads Regularly	A	B	C	$A \cap B$	$A \cap C$	$B \cap C$	$A \cap B \cap C$
Probability	0.14	0.23	0.37	0.08	0.09	0.13	0.05

Then determine the following probabilities $P(A|B)$, $P(A|B \cup C)$ and $P(A \cup B|C)$.

- (c) It is known that 30% of a certain company’s washing machines require service while under warranty, whereas only 10% of its dryers need such service. If someone purchases both a washer and a dryer made by this company, then
- (i) What is the probability that both machines will need warranty service?
 - (ii) What is the probability that neither machine needs service?
3. (a) In a group of five potential blood donors—a, b, c, d, and e—only a and b have type O- positive blood. Five blood samples, one from each individual, will be typed in random order until an O- positive individual is identified. Let the random variable Y = the number of typing necessary to identify an O- positive individual.
- (i) Find the probability mass function (pmf) of Y .
 - (ii) Draw the line graph of pmf.

- (b) A library has an upper limit of 6 on the number of DVDs that can be checked out to an individual at one time. Consider only those who currently have DVDs checked out, and let X denote the number of DVDs checked out to a randomly selected individual. The pmf of X is as follows :

x	1	2	3	4	5	6
P(x)	.30	.25	.15	.05	.10	.15

- (i) Find the variance of X .
- (ii) Find the standard deviation of X .
- (c) Eighteen individuals are scheduled to take a driving test at a particular DMV office on a certain day, eight of whom will be taking the test for the first time. Suppose that six of these individuals are randomly assigned to a particular examiner, and let X be the number among the six who are taking the test for the first time.
- (i) What kind of a distribution X does have (name and values of all parameters)?
- (ii) Compute $P(X = 2)$, $P(X \leq 2)$, and $P(X \geq 2)$.
- (iii) Calculate the mean value and standard deviation of X .
4. (a) A mail-order computer business has six telephone lines. Let A denote the number of lines in use at a specified time. Suppose the pmf of A is as given in the accompanying table :

x	0	1	2	3	4	5	6
P(x)	.10	.15	.20	.25	.20	.06	.04

Calculate the probability of each of the following events:

- (i) {at most three lines are in use}
 - (ii) {fewer than three lines are in use}
 - (iii) {at least three lines are in use}
 - (iv) {between two and five lines, inclusive, are in use.}
- (b) Let A denote the amount of space occupied by an article placed in a 1-ft³ packing container. The pdf of A is

$$f(x) = \begin{cases} 90x^8(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Find cdf of X.
 - (ii) What is $P(X \leq 5)$.
- (c) The article “Expectation Analysis of the Probability of Failure for Water Supply Pipes” (J. of Pipeline Systems Engr. and Practice, May 2012: 36-46) pro- posed using the Poisson distribution to model the number of failures in pipelines of various types. Suppose that for cast-iron pipe of a particular length, the expected number of failures is 1 (very close to one of the cases

considered in the article). Then A , the number of failures, has a Poisson distribution with $\mu = 1$.

(i) Obtain $P(X \leq 5)$.

(ii) Determine $P(2 \leq X \leq 4)$.

(iii) What is the probability that X exceeds its mean value by more than one standard deviation.

5. (a) “Time headway” in traffic flow is the elapsed time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let X = the time headway for two randomly chosen consecutive cars on a freeway during a period of heavy flow. The following pdf of X is essentially the one suggested in “The Statistical Properties of Freeway Traffic” (Transp. Res., vol. 11: 221-228):

$$f(x) = \begin{cases} 0.15e^{-0.15(x-0.5)} & x \geq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that headway time is at most 5 sec.

- (b) Let X , the thickness of a certain metal sheet, have a uniform distribution on $[A, B]$. Write the probability density function (pdf) of X and find the cumulative distribution function (cdf) of X .
- (c) The amount of distilled water dispensed by a certain machine is normally distributed with mean value 64 oz and standard deviation 0.78 oz. What container size c will ensure that overflow occurs only 0.5% of the time?

6. (a) The distribution of egg weights (g) of a certain type is normal with mean value 53 and standard deviation 0.3. Let $X_1, X_2, \dots, X_{12}$ denote the weights of a dozen randomly selected eggs; these X_i 's constitute a random sample of size 12 from the specified normal distribution. Find the mean and standard deviation of the sample mean weight $\bar{X}$. Also, find the probability that the sample mean weight exceeds 53.5 g.
- (b) Medical researchers have noted that adolescent females are much more likely to deliver low-birth-weight babies than are adult females. Because such babies have higher mortality rates, numerous investigations have focused on the relationship between mother's age and birth weight. For the following data on x = maternal age (year) and y = baby's birth weight (kg).

x	15	17	18	15	16	19	17	16	18	19
y	2.3	3.4	3.3	2.6	2.9	3.3	2.9	2.5	3.1	3.6

Compute the coefficient of correlation between x and y . And, how would you describe the nature of relationship between the two variables.

- (c) Consider the following data on fracture strength (x , as a percentage of ultimate tensile strength) and attenuation (y , in neper/cm, the decrease in amplitude of the stress wave) in fiberglass-reinforced polyester composites.

x	12	30	36	40	45	57	62	67	71	78
y	3.3	3.2	3.4	3.0	2.8	2.9	2.7	2.6	2.5	2.6

Find the equation of regression line. And estimate the attenuation (y) when fracture strength (x) is 50.