[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 8061 **L**

Unique Paper Code : 2353010025

Name of the Paper : Integral Equations and Calculus of Variations

Name of the Course : **Bachelor of Science (Honours Course) Mathematics**

Semester : VIII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) Find resolvent kernel for the following integral equation

$$\phi(x) = f(x) + \int_0^x (x-t)\phi(t)\,dt.$$

(b) Solve by successive approximations :

$$\phi(x) = \frac{x^3}{3} - 2x - \int_0^x \phi(t)\,dt, \qquad \phi_0(x) = x^2.$$

(c) Solve the integral equation

$$\phi(x) = 1 - 2x - 4x^2 + \int_0^x [3 - 6(x-t) - 4(x-t)^2]\phi(t)\,dt.$$

P.T.O.

2. (a) Solve convolution equation

$$\phi(x) = \cos(x) - \int_0^x (x-t)\cos(x-t)\,\phi(t)\,dt.$$

(b) Solve the following integral equation of first kind by first reducing it to integral equation of second kind

$$\int_0^x a^{x-t}\,\phi(t)dt = f(x),\ \ f(0) = 0.$$

(c) Solve die integral equation

$$\frac{1}{2}\int_0^x (x-t)^2\phi(t)\,dt = \cos(x) - 1 + \frac{x^2}{2}.$$

3. (a) Show that the function $\varphi(x) = xe^{-x}$ is a solution of the Fredholm-type integral equation

$$\varphi(x) - 4\int_0^1 e^{-(x+t)}\varphi(t)dt = (x-1)e^{-x}.$$

(b) Using Fredholm determinates, find the resolvent kernel of the kernel

$$K(x,t) = \sin(x) - \sin(t);\ \ 0 \le x \le 2\pi, \qquad 0 \le t \le 2\pi.$$

(c) Find the iterated kernels $K_1(x, t)$ and $K_2(x, t)$ if $K(x, t) = e^x \cos(t)$, $a = 0$, $b = \pi$.

4. (a) Solve the integral equation

$$\varphi(x) - \lambda\int_0^1 K(x,t)\varphi(t)dt = x$$

where

$$K(x,t) = \begin{cases} x(t-1), & \text{if } 0 \le x \le t \\ t(x-1), & \text{if } t \le x \le 1 \end{cases}$$

(b) Solve the given Fredholm integral equation

$$\varphi(x) - \lambda \int_0^{\pi} \sin(x-t)\, \varphi(t)dt = \cos(x)$$

where λ is a parameter.

(c) Find the characteristic numbers and eigenfunctions of the integral equation

$$\varphi(x) - \lambda \int_0^{\pi} K(x,t)\varphi(t)dt = 0$$

where

$$K(x,t) = \begin{cases} \cos(x)\sin(t), & \text{if } 0 \le x \le t \\ \cos(t)\sin(x), & \text{if } t \le x \le \pi \end{cases}$$

5. (a) Among all the curves joining two given points (x_0, y_0) and (x_1, y_1), find the one which generates the surface of minimum area when rotated about the x – axis.

(b) Among all the curves lying on the sphere $x^2 + y^2 + z^2 = a^2$ and passing through two given points (x_0, y_0, z_0) and (x_1, y_1, z_1), find the one which has the least length.

(c) Find the shortest distance of point P(0,0,4) from line x = y = z.

6. (a) Find the path on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity.

(b) Among all curves of length 1 in the upper half-plane passing through the points $(-a, 0)$ and $(a, 0)$, find the one which together with the interval $[-a, a]$ encloses the largest area.

(c) Find the shortest distance of point $P(0,2,3)$ from line $x/2 = y/3 = z/4$.