[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 8059 **L**

Unique Paper Code : 2353010023

Name of the Paper : DSE-6(iii): Industrial Mathematics

Name of the Course : **B.Sc. (H) Mathematics (DSE)**

Semester : VIII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. at an appropriate place on receipt of this question paper.
2. Attempt all questions by selecting three parts from Q1.-Q4. and two parts from Q5. - Q6.
3. Parts of the questions to be attempted together.
4. All questions carry equal marks. All parts of a question carry equal marks. Marks are indicated.

Q1. (a) Define the Laplace transform of a continuous time system $f(t)$. Find the Laplace transform of $f(t) = t^n, t \geq 0$, where n is a fixed natural number. (5)

(b) Find the inverse-Laplace transform of $F(s) = \frac{2(s+3)}{(s+1)(s+6)}$. (5)

(c) Define transfer function of a system. Find the transfer function of a system with equation $3\ddot{x} + 4\dot{x} + 5x = p$. (5)

(d) Find the state-space model of a system with equation $\ddot{x} - 2\dot{x} - x = 1$. (5)

Q2. (a) Define signal flow graph of a system. Explain signal flow graph of a parallel system. (5)

(b) Find Z transform of $f(k) = a^k, k \in \mathbb{N}$ and a is a positive constant. (5)

(c) Find the root locus if $G(s)H(s) = \frac{1}{s(s+2)(s^2+4s+20)}$. (5)

(d) Determine whether the characteristic equation $D(s) = s^4 + 4s^3 + 23s^2 + 40s + 50 = 0$ represents a stable system. (5)

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Q3. (a) Define signal power. Find the signal power of $g(t) = 2e^{-\frac{t}{2}}, t \geq 0$. (5)

(b) Find the energy of the signal $g(t) = 5\cos(6t + 45^0)$. (5)

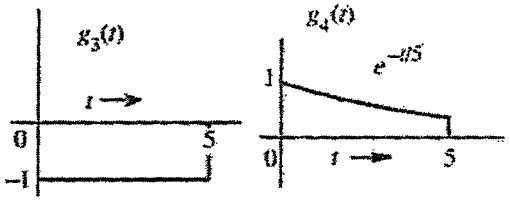
(c) What are analog and digital signals? Give examples. (5)

(d) Consider the signal $g(t) = \begin{cases} t, 0 \leq t \leq 2 \\ 3 - t, 2 \leq t \leq 3 \\ t^2 - 9, 3 \leq t \leq 5 \end{cases}$. Sketch time signals $g(3t)$ and $g(\frac{t}{2})$. (5)

Q4. (a) Briefly explain a unit impulse signal and give one property of unit impulse function. (5)

(b) Among the following six signals, find a pair of orthogonal signals.
(i) $g_1(t) = 2x_1(t) - x_2(t)$
(ii) $g_2(t) = -x_1(t) + 2x_2(t)$
(iii) $g_3(t) = -x_2(t)$
(iv) $g_4(t) = x_1(t) + 2x_2(t)$
(v) $g_5(t) = 2x_1(t) + x_2(t)$
(vi) $g_6(t) = 3x_1(t)$ (5)

(c) Find the correlation coefficient between pulse $g_3(t)$ and $g_4(t) = e^{-t/5}$ given below



(5)

(d) Define orthonormal signals. Give one example in two-dimensional and three-dimensional spaces. (5)

Q5. (a) Obtain the governing equations of the heat absorption model. (7.5)

(b) Find the general solution of the equation $\eta \frac{df}{d\eta} = -3\frac{d^2f}{d\eta^2}$. (7.5)

(c) Show that the substitution $u(x,t) = f(\eta)$, $\eta = xt^{-\frac{1}{2}}$ does not reduce the partial differential equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ to an ordinary differential equation. (7.5)

Q6. (a) Use the transformation $u(x,t) = t^a f(\frac{x}{\sqrt{t}})$, where a is constant and f is a function, to reduce the equation $\frac{\partial u}{\partial t} = b\frac{\partial^2 u}{\partial x^2}$ and show that $af\left(\frac{x}{\sqrt{t}}\right) - \frac{1}{2}\frac{x}{\sqrt{t}}f\left(\frac{x}{\sqrt{t}}\right) = bf''(\frac{x}{\sqrt{t}})$. (7.5)

(b) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, x \geq 0, u(0,t) = 0, u(\infty, t) = 1$. (7.5)

(c) Consider the Fourier series $f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos(\frac{n\pi x}{L}) + \sum_{n=1}^{\infty} B_n \sin(\frac{n\pi x}{L})$. Show that $B_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$. (7.5)

(500)