[This question paper contains 2 printed pages.]

Your Roll No.................

Sr. No. of Question Paper : 8067 **L**

Unique Paper Code : 2353200021

Name of the Paper : Rings and Fields

Name of the Course : **Bachelor of Arts/ Bachelor of Science (Programme)**

Semester : VIII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All questions carry equal marks.

1. (a) Define the kernel of a ring homomorphism. Show that the kernel of a ring homomorphism is an ideal of the domain ring.

   (b) Define ideal of a ring. Prove that the sum of two ideals of a ring is also an ideal.

   (c) Define an Euclidean ring and a principal ideal ring and give one example of each.

2. (a) Let R be a commutative ring with unit element and I be an ideal of R. Prove that I is maximal if and only if the quotient ring R/I is a field.

   (b) Prove that every Principal Ideal Domain is a Unique Factorization Domain.

   (c) State and prove the First Isomorphism Theorem for ring homomorphisms.

3. (a) Prove that the polynomial $f(x) = 1 + x + x^2 + \dots\dots\dots + x^{p-1}$, where p is a prime number, is irreducible over the field of rational numbers.

   (b) Let F be a field. Prove that if $f(x)$ and $g(x)$ are two non-zero element of $F[x]$, then $\deg(f(x)) \leq \deg(f(x).g(x))$.

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(c) Prove that if the primitive polynomial $f(x)$ can be factored as the product of two polynomials having rational coefficient, then it can be factored as the product of two polynomials having integer coefficients.

4. (a) Let F be a field and let $f(x) \in F[x]$. Then prove that any two splitting field of $f(x)$ over F are isomorphic.

   (b) Prove that if a polynomial $f(x)$ over a field F has a multiple zero in some extension E then $f(x)$ *and* $f'(x)$ have a common factor of positive degree in $F[x]$.

   (c) Define Perfect field and give one example of it. Prove that every finite field is Perfect.

5. (a) Prove that every finite extension of a field F is an algebraic extension but the converse may not be true, give example.

   (b) Show that $\sqrt{2} + \sqrt[3]{5}$ is an algebraic element of degree 6 over the field $\mathbb{Q}$.

   (c) Prove that $\mathbb{Q}(\sqrt{3}, \sqrt{5}) = \mathbb{Q}(\sqrt{3} + \sqrt{5})$.

6. (a) If a and b are constructible numbers, give a geometrical proof that ab is constructible.

   (b) Prove that an angle θ is constructible if and only if $\sin\theta$ is constructible.

   (c) Draw the subfield lattice of $GF(2^{24})$ and write down the generator of each subfield.