[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 8066 **L**

Unique Paper Code : 2353200020

Name of the Paper : Optimization Techniques

Name of the Course : **Bachelor of Arts/Bachelor of Science (Programme)**

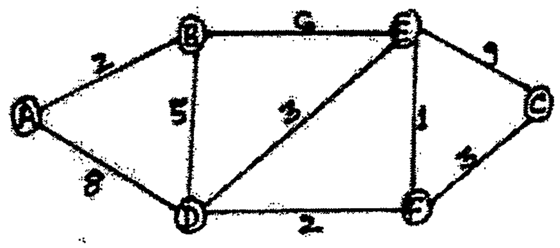
Semester : VIII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator (not scientific) is allowed.

1(a) Find the shortest path from a fixed node A to node C using Dijkstra's algorithm for the following problem



(7.5)

1(b) Solve the following Cost Minimizing Transportation problem

	25	10	20
20	5	1	0
10	3	2	4
15	7	5	2
15	9	6	1

(7.5)

1(c) Find the optimal solution of the fractional knapsack problem with n=7, M=15

Object	1	2	3	4	5	6	7
Wright	2	5	5	9	4	6	3
Profit	4	15	20	18	10	15	12

(7.5)

2(a) Consider a firm having two factories. The firm is to ship products from the factories to three retail stores. The number of units available at factories X and Y are 200 and 300 respectively, while those demanded at retail stores A, B, and C are 100,150 and 200 respectively. Rather than shipping directly from factories to retail stores, it is added to investigate the possibility of transshipment. The transportation cost (in rupees) per unit is given below

		Factory		Retail store		
		X	Y	A	B	C
Factory	X	0	8	7	8	9
	Y	6	0	5	4	3
Retail store	A	7	2	0	5	1
	B	1	5	1	0	4
	C	8	9	7	8	0

Find an optimal shipping schedule. (7.5)

2(b) In a cargo-loading problem, there are three items of different per unit weight and revenue as given below

$Item\ i$	w_i(ton)	r_i($)
1	1	30
2	2	60
3	3	80

Maximum cargo loaded is restricted to 4-ton. The goal is to determine the number of units of each item that will maximize the total return. (7.5)

2(c) What are the essential characteristics of a dynamic programming problem? (7.5)

3 (a) Solve the integer linear programming problem graphically

$$Max\ z = 14x_1 + 16x_2$$

$$subject\ to\ \ 4x_1 + 3x_2 \leq 12$$

$$6x_1 + 8x_2 \leq 24$$

$$x_1, x_2 \geq 0\ \ x_1\ and\ x_2\ integers.$$ (7.5)

3(b) Solve the integer linear programming problem using Gomory's cutting plane methods

$$Max\ z = x_1 + x_2$$
$$subject\ to\ \ 3x_1 + 2x_2 \leq 12\ \ ,$$
$$x_2 \leq 2$$
$$x_1, x_2 \geq 0\ \ \ x_1\ and\ x_2\ \ integers \qquad (7.5)$$

3 (c) Solve the mixed-integer programming problem

$$Max\ z = 7x_1 + 9x_2$$
$$subject\ to\ -x_1 + 3x_2 \leq 6\ ,$$
$$7x_1 + x_2 \leq 35\ ;$$
$$x_1, x_2 \geq 0\ , x_1\ is\ an\ \ integer. \qquad (7.5)$$

4(a) Solve the mixed integer linear programming problem

$$Min\ z = x_1 + x_2$$
$$subject\ to\ \ 3x_1 + 2x_2 \leq 5\ \ ,$$
$$x_2 \leq 2\ \ ;$$
$$x_1, x_2 \geq 0\ \ and\ x_1\ is\ an\ \ integer. \qquad (7.5)$$

4(b) Use branch and bound method to solve the integer linear programming problem

$$Max\ z = 2x_1 + 3x_2$$
$$subject\ to\ \ 5x_1 + 7x_2 \leq 35\ \ ,$$
$$4x_1 + 9\,x_2 \leq 36\ ;$$
$$x_1, x_2 \geq 0\ , x_1\ and\ x_2\ are\ integers. \qquad (7.5)$$

4(c) Use branch and bound method to solve the integer linear programming problem

$$Max\ z = 3x_1 + 4x_2$$
$$subject\ to\ \ 7x_1 + 16x_2 \leq 52\ ,$$
$$3x_1 - 2\,x_2 \leq 9\ ;$$
$$x_1, x_2 \geq 0\ , x_1\ and\ x_2\ are\ integers. \qquad (7.5)$$

5(a) Define concavity of a function. Let S be a non-empty convex set in $\mathbb{R}$. Examine the convexity/concavity of the function $f(x) = e^x$ for $x \in \mathbb{R}$. (7.5)

5(b) Let S be a nonempty convex set of $\mathbb{R}^n$ and let $f : S \rightarrow \mathbb{R}$ and $g : S \rightarrow \mathbb{R}$ be two convex functions. Then, prove that $h(x) = Max(f(x), g(x))$ is a convex function. (7.5)

5(c) Let $S \subseteq \mathbb{R}^2$ be an open convex set and $f : S \rightarrow \mathbb{R}$ be given by

$f(x_1, x_2) = 4x_1{}^2 + x_2{}^2 + 4x_1x_2$, $(x_1, x_2) \in \mathbb{R}^2$. Examine the convexity/strict convexity of f. (7.5)

6(a) Define Convex Programming Problem. Is the following problem a convex programming problem? Give reasons for your answer.

$$Max\ 4x_1 + 3x_2$$

$$\text{subject to } x_1{}^2 + x_2{}^2 \leq 1.$$

Also, show it geometrically. (7.5)

6(b) Solve the following linear fractional programming problem by the Charnes and Cooper Method

$$Max \quad z = \frac{x_2 - 5}{-x_1 - x_2 + 9}$$

$$\begin{aligned} \text{subject to } 2x_1 + 5x_2 &\geq 10 \\ 4x_1 + 3x_2 &\leq 20 \\ -x_1 + x_2 &\leq 2 \\ x_1, x_2 &\geq 0. \end{aligned}$$

(7.5)

6(c) Solve the following linear fractional programming problem by the simplex Algorithm:

$$Max \quad z = \frac{2x_1 + x_2}{-x_1 - 2x_2}$$

$$\begin{aligned} \text{subject to } x_1 + x_2 &\leq 6 \\ 2x_1 + x_2 &\geq 4 \\ x_1, x_2 &\geq 0. \end{aligned}$$

(7.5)

(500)