[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 8064 **L**

Unique Paper Code : 2353200018

Name of the Paper : Elements of Partial Differential Equations

Name of the Course : **DSE-Bachelor of Arts / Bachelor of Science (Programme)**

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Answer any **two** parts from each question.
3. **All** questions carry equal marks.

Q.1 (a) Find the solution of the system of surface which cut orthogonally the cones of the system $x^2+y^2+z^2=cxy$.

(b) Find the characteristics of the equation $pq=z$, and determine the integral surface which passes through the parabola $x=0, y^2=z$.

(c) Show that the equation $z=px+qy$ is compatible with any equation $f(x,y,z,p,q)=0$ that is homogeneous in x, y and z. Solve completely the simultaneous equations $z=px+qy$, $2xy(p^2+q^2)=z(yp+xq)$

Q.2 (a) Find the complete integrals of the partial differential equations $z^2=pqxy$.

(b) Find the solution to the equation $p^2q^2+x^2y^2=x^2q^2(x^2+y^2)$.

(c) Find the integral surface of the differential equation $(y+zq)^2=z^2(1+p^2+q^2)$ circumscribed about the surface $x^2-z^2=2y$.

Q.3 (a) Find the solution of the equations $xp-yq=x$, $x^2p+q=xz$ is compatible and find their solution.

(b) Find the complete integrals of the partial differential equations $(p^2+q^2)x=pz$ and deduce the solution which passes through the curve $x=0, z^2=4y$

(c) Show that a family of spheres $x^2+y^2+(z-c)^2=r^2$, satisfies the partial differential equation $yp-xq=0$

Q.4 (a) Find the solution of $r+2s+t+2p+2q+z=0$

(b) Solve using Monge's method $3s+\left(rt-s^2\right)=2$

(c) Find the solution of $r-s+2q-z=x^2y^2$

Q.5 (a) Find the solution of $\left(D-D'^2\right)z=\cos(x-3y)$

(b) Find the solution of $y^2r-2ys+t=p+6y$

(c) Solve the wave equation $u_{tt}-c^2u_{xx}=0$

with initial conditions $u(x,0)=0,\ u_t(x,0)=1$.

Q.6 (a) Derive the one- dimensional wave equation of a string

$$u_{tt}=c^2u_{xx}+F,$$

where, $c^2=T/\rho$, and T is the tension at the end points

$F=f/\rho$, and f be the external force, acting on the string per unit length.

(b) Determine the solution of wave equation with non-homogeneous boundary value problem:

$$u_{tt}-9u_{xx}=0,\quad x>0,\ t>0,$$
$$u(x,0)=\log(1+x), u_t(x,0)=1,\ x\geq 0,$$
$$u(0,t)=t,\ t\geq 0.$$

(c) Determine the solution of Cauchy problem for non-homogeneous wave equation:

$$u_{tt}=u_{xx}-2,\ x\in\mathbb{R},\ t>0,$$

with initial condition $u(x,0)=\sin x,\ u_t(x,0)=x,\quad x\in\mathbb{R}.$