

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 8063

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Unique Paper Code : 2353200017

Name of the Paper : Applied Algebra

Name of the Course : NEP UGCF-2022 BACHELOR OF ARTS/
BACHELOR OF SCIENCE (PROGRAMME)

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All Questions are compulsory.
3. Attempt any two parts from each question.

1. (a) An economy consists of three sectors: Manufacturing, Agriculture, and Services. The input requirements per unit of output of each sector are given below:

Purchased From/Produced By	Manufacturing	Agriculture	Services
Manufacturing	0.50	0.40	0.20
Agriculture	0.20	0.30	0.10
Services	0.10	0.10	0.30

Suppose the Manufacturing sector plans to produce 100 units of output.

- (i) Write the unit consumption vector for the Manufacturing sector.
- (ii) Determine the intermediate demand vector generated by the Manufacturing sector.
- (iii) Explain briefly how this model illustrates the structure of interdependence in an economy.

- (b) Consider an economy with two sectors having production model as

$$X = Cx + d$$

where C is the consumption matrix given by $\begin{bmatrix} .0 & .5 \\ .6 & .2 \end{bmatrix}$ and d is the final demand vector given by $\begin{bmatrix} 50 \\ 30 \end{bmatrix}$

Use an inverse matrix to determine the production level necessary to satisfy the final demand.

P.T.O.

- (c) Any linear transformation of $\mathbb{R}^2$ can be represented in homogeneous coordinates by a matrix of the form $\begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix}$, where A is a 2×2 matrix.

Explain how homogeneous coordinates are used to represent linear transformations in $\mathbb{R}^2$ and write the homogeneous coordinate matrix for the transformation: Reflection about the line $y = x$.

2. (a) Consider the difference equation:

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k = 0$$

Assume a solution of the form $y_k = r^k$ and derive the auxiliary equation.

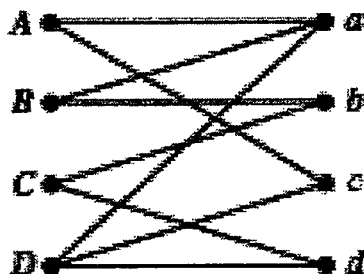
- (b) Consider a Markov Chain with transition matrix

$$P = \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix}$$

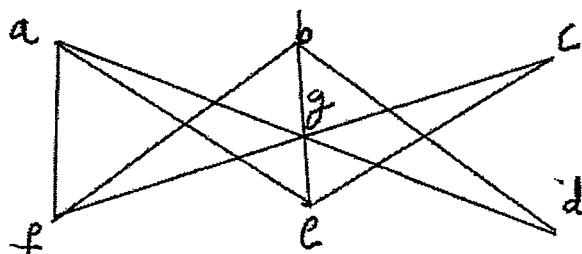
- (i) Compute P^2 and P^3 and describe the pattern observed in the powers of P .
 (ii) Based on the pattern, determine whether the Markov Chain appears to approach a steady-state matrix.
- (c) Determine the least-squares error in the least-squares solution of $Ax = b$ for

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

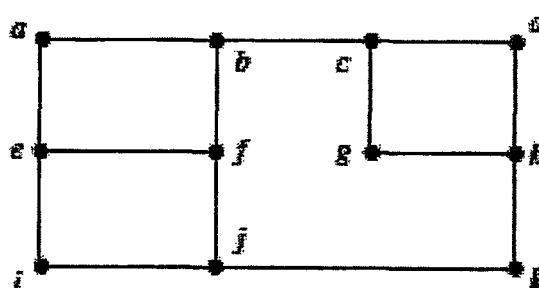
3. (a) Find all matchings of jobs for the following graph:



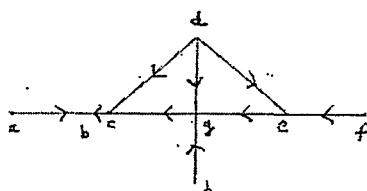
- (b) Prove that the table for any finite group $(G, +)$ of order n is a Latin square of order n based on G .
- (c) Find a minimal edge cover for the following graph. Give a detailed logical analysis.



4. (a) Find a largest independent set in the graph:



- (b) Find a vertex basis for the following graph. Justify your answer.



- (c) A supermarket wishes to test the effect of putting cereals on four shelves at differential heights, show how to design such an experiment lasting four weeks and using four brands of cereals.
5. (a) Define the 14 digit Airline Ticket check digit scheme. Explain the cases where the Airline Ticket check digit scheme fails to catch single-digit errors.
- (b) Using the ISBN check digit scheme, determine which of the following is a valid ISBN and which is invalid.
 $0 - 201 - 52032 - X$ and $3 - 357 - 02001 - 4$

- (c) Define the 12-character Washington State Driver's License number check digit scheme. Create a Washington State Driver's License number, complete with check digit, for Joseph Marshall Pollan born on October 19, 1970.
6. (a) Explain mathematically how the UPC check digit scheme catches all single-digit errors.
- (b) Define a regular polygon. Examine the twelve symmetries of a hexagon in detail.
- (c) Define Glide Reflective rigid motion. Explain the generation of the pattern below by using rigid motion:

