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Your Roll No.------------------------

Serial Number of Question Paper	:	**8118**
Unique Paper Code	:	**2223010038**
Name of the Paper	:	**Adv. Quantum Mechanics-II**
Name of the Course	:	**B.Sc. Hons (Physics) NEP: UGCF-2022**
Semester	:	**VIII- Semester (DSE Paper)**

Maximum Marks: 90

Duration : 3 Hours

<u>Instructions</u>

1. Write your Roll Number on the top immediately on the receipt of the question paper.
2. Attempt five questions in all. Question number 1 is compulsory. Attempt any four questions amongst questions 2-6.
3. All questions carry equal marks.
4. Use of non-programmable scientific calculators is allowed.

1. Attempt any 6 of the following : **[6×3=18]**

   a) For a linear harmonic oscillator, the force constant changes from k to $k+\lambda$, where $\lambda \ll k$. Treat the change in force constant as a perturbation and write the perturbation Hamiltonian.

   b) A variational trial wave function gives expectation energy values $6eV$, $5eV$, and $4.5eV$ for three different choices of the variational parameters. Which value gives the best variational estimate of ground-state energy? Justify your answer.

   c) A perturbation acts on a quantum system for time interval δt. Determine whether the evolution is sudden or adiabatic if $\delta t \gg \hbar/\delta E$ where δE is a characteristic energy difference of the system. Justify your answer.

   d) A particle in a box of length L suddenly expands to length $2L$. If initially, the particle was in the ground state, what is the probability that after the expansion, it would still be in the new ground state?

   e) In the first-Born approximation, the scattering amplitude is $f(\theta) = \frac{A}{1+\cos\theta}$ where $A = 2 \times 10^{-11}$ m. Calculate the differential cross-section at $\theta = 30°$.

   f) Explain the meaning of the scattering matrix (S-matrix) in quantum scattering theory. What physical information is obtained from its diagonal elements?

g) For a scattering process, the forward scattering amplitude is $f(0) = (2 + i5) \times 10^{-11}$ m. The wave number of the incident particle is $k = 2 \times 10^{10}\text{m}^{-1}$. Using the optical theorem, calculate the total scattering cross-section.

2.

a) Consider a quantum system with Hamiltonian $H = H_0 + \lambda V$, where H_0 has a complete set of non-degenerate eigenstates. Using time-independent perturbation theory, derive the expressions for the first-order correction to the energy and the first-order correction to the eigenfunction. [9]

b) Consider the one-dimensional quantum anharmonic oscillator described by the Hamiltonian $H = H^{(0)} + H'$ with $H^{(0)} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$ and $H' = \lambda\hat{x}^4$ a small perturbation. Using a characteristic length scale $d = \sqrt{\frac{\hbar}{m\omega}}$ and the ladder operator representation of the position operator, $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$, express the perturbation in terms of the creation and annihilation operators. Using first-order time-independent perturbation theory, compute the correction to the ground state energy due to the perturbation. [9]

3

a) Combine the relativistic and spin--orbit corrections for the hydrogen atom to obtain the fine-structure energy correction. Show that the corrected energy levels depend only on the principal quantum number n and the total angular momentum quantum number j. What is the physical significance of the remaining degeneracy of states having the same values of n and j? **[9]**

b) Starting from the Lippmann-Schwinger equation, derive the first-Born approximation for the scattering amplitude $f(\theta, \phi)$. Show that, for a central potential $V(r)$, the scattering amplitude depends only on the scattering angle θ and can be written as

$$f(\theta) = -\frac{2m}{\hbar^2 q}\int_0^\infty r\,V(r)\sin(qr)\,dr$$

where $q = |\vec{k_f} - \vec{k_i}|$ is the momentum transfer. State the conditions for the validity of the Born approximation. **[9]**

4.

a) Using the interaction picture, derive the coupled differential equations for the time evolution of the expansion coefficients in the time-dependent perturbation theory. Hence obtain the first-order transition amplitude from an initial state to a final state. State the conditions under which higher-order terms in the perturbation expansion may be neglected. **[9]**

b) The Yukawa potential describing the nucleon-nucleon interaction is given by,

$$V(r) = -r_0 V_0 \frac{e^{-r/r_0}}{r}$$

where r is the distance between the nucleons, and r_0 is the range related to the mass of the meson (m_π) by $r_0 = \frac{\hbar}{m_\pi c}$. Using the normalized trial wavefunction $\psi(r) = \sqrt{\frac{\beta^3}{\pi r_0^3}}\, e^{-\frac{\beta r}{r_0}}$, calculate the variational estimate of the ground-state energy of the proton--neutron system. Obtain the condition under which the Yukawa potential can support the existence of the deuteron (a bound state of the proton and the neutron)? **[9]**

5.

a) A two-level quantum system, initially in its ground state $|1\rangle$, is subjected to a weak time-dependent perturbation

$$V(t) = V_0 \sin(\omega t), \qquad 0 < t < T,$$

where V_0 is a constant. The matrix element between the two states is given by $\langle 2|V_0|1\rangle = V_{21}$. In the first-order time-dependent perturbation theory near resonance, the transition probability $P_{1\to 2}(T)$ may be approximated as

$$P_{1\to 2} \cong \frac{|V_{21}|^2}{\hbar^2}\left[\frac{\sin\left\{\frac{(\omega_{21}-\omega)T}{2}\right\}}{\omega_{21}-\omega}\right]^2, \quad \omega_{21} = \frac{(E_2 - E_1)}{\hbar}, \quad \hbar = 1.05\ \times 10^{-34}\ \text{Js}$$

Given that $V_{21} = 2.0 \times 10^{-23}$ J and the perturbation is applied for time $T = 1.5 \times 10^{-6}$ s, calculate the resonant angular frequency ω_{21} and the transition probability at exact resonance. Estimate $\Delta\omega = \omega - \omega_{21}$ for which the transition probability first becomes zero. **[9]**

b) Using partial wave analysis of scattering, derive the expression for scattering amplitude and the total scattering cross-section in terms of phase shifts δ_l. For s-wave scattering only ($l = 0$) with phase shift $\delta_0 = \pi/6$, calculate the total cross-section. **[9]**

6. Consider scattering of a particle of mass m from an attractive spherical square-well potential

$$V(r) = \begin{cases} V_0 \text{ for } r \le a \\ 0 \text{ for } r > a \end{cases}, \quad V_0 < 0\,.$$

a) Using partial wave analysis for scattering states ($E > 0$), obtain the matching condition that determines the s-wave ($\ell = 0$) phase shift δ_0. shift δ_0. Hence show that, at high energies, $\delta_0(k) = maV_0/\hbar^2$, where $k = \sqrt{2mE}/\hbar$. **[9]**

b) Show that the above high energy result for $\delta_0(k)$ also follows from the Born approximation. **[9]**