[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 10048 **L**

Unique Paper Code : 2353200018

Name of the Paper : Elements of Partial Differential Equations

Name of the Course : **DSE- Bachelor of Arts/Bachelor of Science (Programme)**

Semester : VIII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Answer any **two** parts from each question.
3. **All** questions carry equal marks.

Q.1 (a) Find the solution if u is function of x, y and z and which satisfies the partial differential equation $(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$

(b) Find the general solution to the partial differential equation $px(x+y)=qy(x+y)-(x-y)(2x+2y+z)$

(c) Verify that the differential equation $z(z+y)dx+z(z+x)dy-2xydz=0$ is integrable and find its primitive.

Q.2 (a) Find the surface which intersects the surfaces of the system $2y(z-3)p+(2x-z)q=y(2x-3)$. Orthogonally and which passes through the circle $z=0, x^2+y^2=2x$

(b) Find the solution to the equation $a^2y^2z^2dx+b^2z^2x^2dy+c^2x^2y^2dx=0$

(c) Find the characteristics of the equation $z=p^2-q^2$ which passes through the parabola $4z+x^2=0, y=0$.

Q.3 (a) finds the solution of the equations $xp-yq=x$, $x^2p+q=xz$ is compatible and find their solution.

(b) Find the complete integrals of the equations $xp+3yq=2(z-x^2q^2)$

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(c) Find the solution of the equation $\nabla_1^2 z = e^{-x}\cos y$ which tends to zero as $x \to \infty$ and has the value $\cos y$ when $x = 0$.

Q.4 (a) Show that the family of spheres $(x-a)^2+(y-b)^2+z^2=r^2$ satisfies the partial differential equation $z^2\left(p^2+q^2+1\right)=r^2$

(b) Find the solution of $4r-4s+t=16\log(x+2y)$

(c) Solve by Monge's method: $2x^2r-5xys+2y^2t+2\left(px+qy\right)=0$

Q.5 (a) Find the solution of $(D^3-4D^2D'+4DD'^2)z=4\sin(2x+y)$

(b) Find the solution of : $r+t-\left(rt-s^2\right)=1$

(c) Determine the solution of the initial-value problem for an infinite string with initial condition $u_{tt}-c^2u_{xx}=0,\quad x\in\mathbb{R},\ t>0,$

$$u(x,0)=f(x),\ x\in\mathbb{R}$$

$$u_t(x,0)=g(x),\ x\in\mathbb{R}$$

Q.6 (a) Determine the solution of the initial-value problem for a semi-infinite string with fixed end $u_{tt}-16u_{xx}=0,\quad 0<x<\infty,\ t>0,$

$$u(x,0)=\sin x,\ 0\le x<\infty$$

$$u_t(x,0)=x^2,\ 0\le x<\infty$$

$$u(0,t)=0,\qquad 0\le t<\infty$$

(b) Determine the solution of wave equation with non-homogeneous boundary -value problem $u_{tt}-9u_{xx}=0,\quad x>0,\ t>0,$

$$u(x,0)=\cos x,\ x\ge 0,$$

$$u_t(x,0)=2,\ x\ge 0,$$

$$u_x(0,t)=1-t,\ t\ge 0$$

(c) Determine the solution of Cauchy problem for non-homogeneous wave equation:

$u_{tt}=u_{xx}\ x\in\mathbb{R},\ t>0,$

with initial condition $u(x,0)=\sin x,\ u_t(x,0)=x,\quad x\in\mathbb{R}.$

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