[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 8139 **L**

Unique Paper Code : 2373010021

Name of the Paper : Advanced Stochastic Processes

Name of the Course : **B.Sc. (H) Statistics**

Semester : VIII (NEP-UGCF)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all. Question no. **1** is compulsory.
3. Attempt **five** questions from the remaining questions.
4. Use of non-programmable scientific calculator is allowed.

Q1 Attempt any *three* parts

(a) What is the meaning of the Inverse Laplace transform of a function? Explain in detail by an example.

(b) What is a Stochastic Matrix? What is the relation between a Markov Matrix and a Stochastic Matrix?

(c) What do you mean by Spectral Representation of a matrix?

(d) What do you understand by the limiting behaviour of an aperiodic chain?

(3x5=15)

Q2 (a) Write a detailed note on determining higher transition probabilities of a Markov Chain.

(b) A Markov chain $\{X_n, n \geq 0\}$ has state space $S = \{1,2,3,4\}$ with the following transition probability matrix, determine $\lim_{n\to\infty} p_{ij}^{(n)}$, for all $i, j \in S$.

$$P = \begin{bmatrix} 1/4 & 3/4 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 2/5 & 3/5 \end{bmatrix}.$$

(7, 8)

Q3(a) Explain the graph-theoretic approach to classifying states of a Markov chain. How do you identify communicating classes from a directed graph?

(b) Consider the three state Markov chain with the following transition probability matrix

$$P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

Obtain the limiting distribution using graph theoretic approach. (7, 8)

Q4. What is a 'Linear-Growth Process'? Obtain the generating function and mean population size for this process. (15)

Q5(a) Define G.W. branching process assuming that the process starts with a single ancestor.

(b) Let the distribution of the number of off springs for the above case be geometric with: $p_k = P(X = k) = b(1-b)^k, k = 0, 1, 2, \ldots\ldots (0 < b < 1)$. Obtain the probability generating function (p.g.f.) of the offspring distribution and hence obtain the probability of ultimate extinction.

(3, 12)

Q6(a) In the context of renewal processes write a detailed note on 'Renewal Interval'

(b) Let the genotypes AA, Aa, aa be denoted by 1, 2, 3 respectively. Let $X_n, (n \geq 1)$ be the genotype of the offspring of a father of genotypes X_{n-1}. Let $\{X_n,\ n \geq 0\}$ be a Markov chain with state space $S = \{1,2,3\}$ and transition matrix

$$P = \begin{bmatrix} p & q & 0 \\ p/2 & 1/2 & q/2 \\ 0 & p & q \end{bmatrix}; p + q = 1, 0 < p < 1.$$

Obtain the limiting distribution of the genotypes.

(5,10)

Q7 (a) Explain the Laplace-Stieltjes transform of a probability distribution. Compute moments in terms of Laplace transform, and hence, find the mean and variance.

(b) Arrival at a telephone booth is considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 minutes. What is the probability that a person arriving at the booth will have to wait?

(10, 5)