[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 8142 **L**

Unique Paper Code : 2373570010

Name of the Paper : DSE: Multivariate Data Analysis

Name of the Course : **B.A. / B.Sc. (P) under NEP-UGCF**

Semester : VIII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all, selecting **Three** from each section.
3. Use of non-programmable scientific Calculator allowed.

SECTION A

1. (a) For the bivariate normal distribution :

$$f(x,y) = kexp(-\frac{1}{102}[(x+2)^2 - 2.8(x+2)(y-1) + 4(y-1)^2]);$$

obtain (i) the parameters of the distribution (ii) the mean and variance of the conditional distribution of Y|X=5.

(b) For $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, prove that X and Y are independent iff $\rho = 0$. (8,7)

2. (a) Derive the moment generating function for $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$.

(b) If X_1 and X_2 are standard normal variates with correlation coefficient ρ between them then show that correlation coefficient between X_1^2 and X_2^2 is ρ^2. (8,7)

3. (a) If X and Y are standard normal variates with coefficient of correlation ρ between them, then obtain the distribution of

$$Q = \frac{X^2+Y^2-2\rho XY}{1-\rho^2}.$$

(b) If $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, then find the correlation coefficient between e^X and e^Y. (8,7)

4. (a) If $(X,Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ then show that linear combination of X and Y is a normal variate. Prove that converse also holds good.

(b) If $\underset{\sim}{X}$ have a p-variate normal distribution $N_p(\underset{\sim}{\mu}, \Sigma)$ and D is $p \times p$ non-singular matrix then show that $D\underset{\sim}{X} \sim N_p(D\mu, D\Sigma D')$. (8,7)

SECTION B

5. (a) If $\underset{\sim}{X}$ have a *p-variate* normal distribution $N_p(\underset{\sim}{\mu}, \Sigma)$ and if $\underset{\sim}{X} = (\underset{\sim}{X_1}, \underset{\sim}{X_2})$, where $\underset{\sim}{X_1}$ is a $(q \times 1)$ and $\underset{\sim}{X_2}$ is a $((p-q) \times 1)$ sub-vector, obtain conditional distribution of $\underset{\sim}{X_1}$ given $\underset{\sim}{X_2} = \underset{\sim}{x_2}$. Hence show that (i) the mean of $\underset{\sim}{X_1}$ given $\underset{\sim}{X_2} = \underset{\sim}{x_2}$ is linear function of given $\underset{\sim}{X_2}$ (ii) covariance matrix of $\underset{\sim}{X_1}$ given $\underset{\sim}{X_2} = \underset{\sim}{x_2}$ does not dependent on $\underset{\sim}{x_2}$.

(b) Let X_1, X_2 and X_3 are three variables measured from their respective means, obtain the equation of regression plane X_3 on X_1 and X_2. (8,7)

6. (a) Derive the moment generating function of $\underset{\sim}{X}$, where $X \sim N_p(\underset{\sim}{\mu}, \Sigma)$.

(b) Let $\underset{\sim}{X}$ follows $N_3(0, \Sigma)$ with $\Sigma = \begin{bmatrix} 1 & \rho & 0 \\ \rho & 1 & \rho \\ 0 & \rho & 1 \end{bmatrix}$, then show that $\rho^2 < \frac{1}{2}$. Further, obtain the value of ρ such that $(X_1 + X_2 + X_3)$ and $(X_1 - X_2 - X_3)$ are independent. (8,7)

7. (a) Explain the concept of Principal components Analysis? Discuss the properties of Principal Components. What are its uses.

(b) Obtain principal component of

$$\Sigma = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & -2 & 5 \end{bmatrix}.$$

Also, calculate the proportion of the total population variance explained by the first principal component. (6,9)

8. Describe an orthogonal factor model stating clearly the underlying assumption. Prove that the covariance structure of such a model is expressed by $\Sigma = LL' + \varphi$, defining clearly the i[th] communality and the specific variance. (15)