

[This question paper contains 2 printed pages.]

Your Roll No.....
आपका अनुक्रमांक.....

Sr. No. of Question Paper : 15172

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Unique Paper Code : 2272101202

Name of the Paper : Intermediate Mathematical Methods for Economics

Name of the Course : ITEP

Semester : II, 2025-2026

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **Any Five** questions.
3. The use of scientific calculator is allowed.
4. Answers may be written either in English or Hindi; but the same medium should be used throughout the paper.

छात्रों के लिए निर्देश

1. इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए।
2. किन्हीं पांच प्रश्नों के उत्तर दीजिये।
3. वैज्ञानिक कैलकुलेटर का उपयोग करने की अनुमति है।
4. इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए।

(5×18=90)

1. (a). Find the values of k for which the following system of equations has unique solution, no solution or infinitely many solutions

$$\begin{aligned}2x + ky + z &= 2 \\4x + 2ky + kz &= 4 \\x + 2y + 3z &= 6\end{aligned}$$

- (b) For the following 2×2 matrix $A = \begin{pmatrix} 3 & 2 \\ 2 & -2 \end{pmatrix}$. Find A^{30} .

- (c) Find a linear approximation of the function $f(x,y) = \frac{1}{1+x-y}$ at $(2,1)$

(6+6+6)

2. (a) Consider four linearly independent vectors u, v, w and $x \in \mathbb{R}^4$. Are the vectors $(u+v), (v+w), (w+2u)$, and $(-2v+u)$ linearly independent or dependent

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- (b) Find the equation of a hyperplane through the origin in the direction of the vectors $(1,2,2)$ and $(4,1,-1)$. Where does the above plane intersect the line given by $x_1 = -t + 2$, $x_2 = 2t - 1$ and $x_3 = t + 3$
- (c) Find numbers a and b that makes A the inverse of B when
- $$A = \begin{pmatrix} 2 & -1 & -1 \\ a & 1/4 & b \\ 1/8 & 1/8 & -\frac{1}{8} \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 6 \\ 1 & 3 & 2 \end{pmatrix}$$
- (6+6+6)
3. (a) In a competitive market, a firm produces good Q according to the function $Q(K, L) = 8K^{1/2}L^{1/4}$. Where, K and L are capital and labour respectively. Given the unitary prices of Rs.5 for output and Rs.2 and Rs.10 for inputs, Find the optimal values of K and L which maximise the profit π . Also, calculate the maximum profit, π .
- (b) The function F is defined for all x and y by $F(x, y) = xe^{y-3} + xy^2 - 2y$. Show that the point $(1,3)$ lies on the level curve $F(x, y) = 4$. And find the equation of the tangent line to the curve at the point $(1,3)$
- (c) Suppose A is positive semidefinite and symmetric. Prove that A is positive definite if $|A| \neq 0$ (6+6+6)
4. (a) suppose a monopolist is practicing price discrimination in the sale of a product by charging different prices in two separate markets. Suppose the demand curves are $P_1 = 100 - Q_1$ and $P_2 = 80 - Q_2$. And suppose that the cost function is $C = 6(Q_1 + Q_2)$. how much should be sold in the two markets to maximize profits? What are the prices charged? How much profit is lost if price discrimination is made illegal?
- (b) An insect is crawling on a hot surface the temperature of which at the point x units to the right of the lower left corner and y units up from the lower left corner is given by $T(x, y) = 100 - x^2 - 3y^3$. If the insect is at the point $(2,1)$, in what direction should it move to cool off the fastest? How fast will the temperature drop in this direction.
- (c) Compute the elasticity of substitution for the Cobb-Douglas function $F(K, L) = AK^a L^b$ (6+6+6)
5. (a) Are the following functions homogeneous? If yes, what is the degree of homogeneity? Also check if they are homothetic.
- i. $F(x, y) = \frac{x^2 + y^2}{xy}$
- ii. $g(x, y) = 2 \ln x - \ln y$
- (b) Consider the function $f(x, y) = ax^2y + bxy + 2xy^2 + c$. Determine the values of the constant a, b and c such that f has a local minimum at the point $(2/3, 1/3)$ with local minimum value of $-1/9$.
- (c) Let $f(x, y) = 3x^2y + 2y^3$. Then $f(1, -1) = -5$. Estimate the value of $f(0.98, -1.01)$. How large is the error caused by this approximation. (6+6+6)
6. (a) Classify the stationary points of $f(x, y) = (x - 2)e^{x^2 - x}e^{(y - 2)^2}$
- (b) Determine the concavity or convexity of the following function. Also check for quasi-concavity and quasi-convexity.
- $$f(x, y, z) = x^2 + y^2 + 3z^2 - xy + 2xz + yz$$
- (c) For $a, b \in \mathbb{R}^n$. Prove that $\|a + b\| = \|a - b\|$, if and only if a and b are orthogonal. (6+6+6)