[This question paper contains 4 printed pages.]

Your Roll No...............

आपका अनुक्रमांक............

Sr. No. of Question Paper : 15207 **L**

Unique Paper Code : 2272102403

Name of the Paper : Introductory Econometrics

Name of the Course : **ITEP**

Semester : IV, 2025-2026

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Use of simple non-programmable calculator is allowed. Statistical tables are attached for your reference
3. Answers may be written either in English or Hindi; but the same medium should be used throughout the paper.

छात्रों के लिए निर्देश

1. इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए।
2. साधारण नॉन-प्रोग्रामेबल कैलकुलेटर का उपयोग अनुमत है। सांख्यिकीय सारणियाँ आपके संदर्भ के लिए संलग्न हैं।
3. इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए।

Answer any Five questions out of Seven. All questions carry equal marks.

1. State whether the following statements are True or False. Give reasons for your answer.
   (i) In order for OLS estimators to possess the Best Linear Unbiased Estimator (BLUE) property, the assumptions of the Classical Linear Regression Model (CLRM) must be satisfied.
   (ii) The Jarque-Bera test for normality is based on the skewness and kurtosis of the OLS residuals, and under the null hypothesis of normality, the test statistic follows a chi-square distribution with 2 degrees of freedom.
   (iii) In cases of high multicollinearity, it is not possible to assess the individual significance of one or more partial regression coefficients.
   (iv) Heteroscedasticity causes OLS to consistently underestimate the standard errors of the regression coefficients.
   (v) The Durbin-Watson d test assumes that the variance of the error term is homoscedastic.
   (vi) If an omitted variable is uncorrelated with the included regressor, the OLS estimator of the included regressor's coefficient will still remain unbiased even though the variable is left out of the model. (3x6=18)

2. a) You are given the following OLS regression results based on a time-series sample:

Model I:

$\hat{Y}_t = 9734.2 - 3782.2X_{2t} + 2815X_{3t}$

$t = (3.3705)\ (-6.6070)\ (2.9712)$, $R^2 = 0.7706$, $\bar{R}^2 = 0.7353$

Model II:

$\hat{Y}_t = 16899 - 2978.5X_{2t}$

$t = (8.5152)\ (-4.7280)$, $R^2 = 0.6149$

(i) Can you find the sample size (n) underlying these results?

(ii) Test between the restricted and unrestricted regression at 10% level of significance.

(b) The estimated wage equation for 200 individuals is as follows:

$\widehat{Wage}_i = -7.09 + 1.9339\ Education_i + 0.6493\ Experience_i$

se: (1.85) (0.4200) (0.1550)

$R^2 = 0.194$, $n = 200$

To test for heteroskedasticity, an auxiliary regression is estimated using the squared residuals from the above model as the dependent variable. The independent variables in this auxiliary regression include education, experience, the square of education, the square of experience, and the cross product of experience and education. It was found that the R-square of this auxiliary regression is 0.0614. Test for heteroscedasticity at 5% level of significance. (9×2=18)

3. (a) The monthly demand for electricity (in units) in a city is given by the following regression output:

$\hat{Y}_i = 310.45 + 6.218X_{2i} + 22.410X_{3i}$

se *(58.32)* *(4.015)* *(10.320)*

$R^2 = 0.76$ $n = 25$

Where Y = Monthly electricity demand (in units)

X_2 = Average temperature (in °C)

X_3 = Number of electrical appliances owned

(i) Test at the 5% level whether X_2 has a significant positive effect on electricity demand.

(ii) Test whether X_3 has a significant positive effect at the 5% level.

(iii) Assess the overall significance of the regression model using the F-test at the 5% level.

(b) Consider the following sample data (in deviation form: $x_i = X_i - \bar{X}$, $y_i = Y_i - \bar{Y}$):
$\Sigma x_i^2 = 200$, $\Sigma y_i^2 = 450$, $\Sigma x_i y_i = 180$, $n = 10$
Derivation of OLS estimators from the objective of minimising the Sum of Squared Residuals ($SSR = \Sigma \hat{e}_i^2$). Calculate $\hat{\beta}_2$, $Var(\hat{\beta}_1)$, and $SE(\hat{\beta}_2)$, given $\sigma^2 = 5.?$
(9x2=18)

4. (a) Consider the following multiple regression output:
$\hat{Y}_i = 720.6 + 18.3X_{2i} + 21.7X_{3i}$
se: (145.30) (10.25) (9.80)
$R^2 = 0.83$ $n = 30$

Where Y = Monthly salary of a software engineer ($)
X_2 = Years of experience in the current company
X_3 = Total years of work experience

If the R^2 of the auxiliary regression between X_2 and X_3 is 0.85:

(i) Calculate the VIF for X_2 (and X_3). What does this indicate?
(ii) Calculate the 95% confidence interval for β_2 and β_3. Are they individually significant at 5%?
(iii) Conduct an F-test for the joint significance of X_2 and X_3 at $\alpha = 5\%$.

(b) Briefly explain different patterns and forms of heteroskedasticity typically found in cross-sectional data sets. Give two real-world examples. (9×2=18)

5. (a) What are the problems of multicollinearity, and what are its remedies?
(b) For a particular cafeteria, the following equation was estimated using yearly data on cups of chocolate-flavoured cold coffee sold:
$\log(Q_t) = 1.534 - 0.250\log(P_t) + 0.025\log(P_t^{**})$
se = (0.2001) (0.240) (0.016)
$R^2 = 0.804$ TSS = 4803 (t = 1991–2023)

Where:
Q_t = Cups of chocolate-flavoured cold coffee sold
P_t = Price per cup of chocolate-flavoured cold coffee (in rupees)
P_t^{**} = Price per cup of vanilla-flavoured cold coffee (in rupees)
Note: All cups in the cafeteria are of the same size.
(i) What can you say about the own-price and cross-price elasticity of demand for chocolate-flavored coffee?
(ii) Test the hypothesis that the own-price elasticity of demand for chocolate-flavoured coffee is unitary elastic at 10% level of significance.
(iii) Draw the ANOVA table for the regression equation. Also calculate R^2.
(6+12=18)

6. (a) Consider the following estimated earnings model:
$\widehat{Earnings}_i = 4.20 + 0.28Age_i - 0.0018Age_i^2 + 5.10Education_i$
se: (1.85) (0.085) (0.0006) (1.20)
$R^2 = 0.82$, $n = 30$

(i) Why is the age coefficient positive, and why is the age square coefficient negative?

(ii) Compute predicted earnings for a person aged 30 with 12 years of education. Also predict for age 40, same education. By how much do earnings change?

(iii) Interpret the coefficient of Education. If a person has 16 years of education instead of 12, what is the predicted difference in earnings (at age 30)?

(iv) Test the overall significance of the model.

(b) A study examines monthly sales (Y, Rs. '000) for male- and female-owned retail shops, with D = 1 for male-owned and D = 0 for female-owned, and X = years of operation. N=15

The combined estimated model is: $\hat{Y} = 50 + 3X + 20D + 1.5(D \times X)$

(i) Interpret each coefficient. What types of regressions does this represent (parallel, concurrent, coincident, or dissimilar)?

(ii) Predict sales for a female-owned shop with 5 years of operation and a male-owned shop with 5 years. By how much do they differ?

(iii) The standard error of the coefficient of the interaction term $(D \times X)$ is $se(\hat{\beta}_4) = 0.65$. Test whether the slope of years of operation differs significantly between male-owned and female-owned shops at $\alpha = 5\%$. (9×2=18)

7. (a) The Cobb-Douglas production function is estimated as:

$\ln Q_i = \beta_0 + \beta_1 \ln L_i + \beta_2 \ln K_i + \varepsilon_i$

The OLS estimates obtained from a sample of n = 25 firms for each industry are reported below:

Industry	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	R^2
Cotton	1.12	0.78	0.26	0.95
Sugar	2.45	0.55	0.38	0.82

(i) Interpret the partial elasticities for both industries. Determine whether each industry exhibits increasing, decreasing, or constant returns to scale.

(ii) If labour increases by 10% in the Cotton industry (holding K constant), what is the percentage change in output?

(iii) Test $H_0: \beta_1 + \beta_2 = 1$ for the Cotton industry, given $Var(\hat{\beta}_1) = 0.04$, $Var(\hat{\beta}_2) = 0.01$, $Cov(\hat{\beta}_1, \hat{\beta}_2) = -0.01$. Use $\alpha = 5\%$.

(b) In estimating the share of agriculture in a country's value added, the following results were obtained from annual data for the years 1999 to 2014:

$\hat{Y}_t = 0.4529 - 0.0041\,t$

$R^2 = 0.5284$, DW = 0.8252

where Y = agricultural share in value added and t = time trend.

(i) Test for the presence of first-order serial (autocorrelation at the 5% level of significance using the Durbin-Watson test. State the null and alternative hypotheses, the decision rules (dL and dU from the table), and your conclusion. (9×2=18)